FUZZY STEINER μ DIMENSION AND EMBEDDING THEOREM

G. PRISCILLA PACIFICA[1], J. JENIT AJITHA [2]

[1] Assistant Professor, Department of Mathematics, St.Marys' College (Autonomous), Thoothukudi. Tamilnadu, India.
Mail Id: priscillamelwyn@gmail.com

[2] Research Scholar(Reg No: 19222212092011), Department of Mathematics, St.Marys' college (Autonomous), Thoothukudi, Affliated by Manonmanium Sundaranar University, Tirunelveli, Tamil Nadu, India.
Mail Id: ajitha.jenit@gmail.com

ABSTRACT
For a connected fuzzy graph G, the Steiner μ-distance of any two nodes of a non-empty set S ⊆ V (G) is defined as the minimum of sum of reciprocals of arc weights of minimum connected fuzzy sub graphs containing S. These fuzzy sub graphs are called fuzzy Steiner trees for S. In this article fuzzy Steiner μ Dimension is introduced and its properties are analysed.
AMS Subject Classification 2010 : 05C72, 05C69, 51E10

Key words : Steiner μ dimension, Steiner μ basis, Steiner μ resolving set, Steiner μ representation

1. INTRODUCTION

Fuzzy graph theory was developed by Rosenfeld in 1975 and has enumerous real life applications. The fuzzy analog of several graph theoretic concepts and strong arcs in fuzzy graphs was elucidated by Rosenfeld [2]. Also he defined some metric aspects using the μ-distance in fuzzy graphs [7]. Distance in graphs was analysed by Harary and Buckley [3]. In this article, the Steiner μ basis is defined for a connected fuzzy graph and its fuzzy cardinality is termed as the steiner μ dimension.

2. Priliminaries

The following are the fundamental definitions which are necessary for this article. The fuzzy Steiner tree of a non empty subset of nodes S of a connected fuzzy graph G(V,σ,μ) is defined as the minimal connected fuzzy subgraphs whose node set contains S. The fuzzy Steiner interval, FI(S) of a non empty subset of nodes S is defined by FI(S) = {u ∈ V (G) / u lies on a fuzzy Steiner tree for S in G}. The fuzzy Steiner μ-distance between any two nodes of a non-empty set S ⊆ V (G) is defined as the minimum of sum of reciprocals of arc weights of minimum fuzzy Steiner tree of S. The fuzzy Steiner μ-distance of S is denoted by $d_{\mu G}(S)$ (or) $d_{\mu S}(u,v)$ where u and v are nodes in S. The fuzzy Steiner μ_k-eccentricity $e_{\mu_k G}(u)$ of a node u in V (G) is given by $e_{\mu_k G}(u) = \max\{d_{\mu S}(u,v)/ S \subseteq V(G), |S| = k \,\&\, u, v \in S\}$.

The fuzzy Steiner μ_k radius of any node u in G is given by $r_{\mu_k G}(G) = \min\{e_{\mu_k G}(u)/ u \in V(G)\}$. The fuzzy Steiner μ_k diameter of a node u in V (G) is given by $diam_{\mu_k G}(G) = \max\{e_{\mu_k G}(u)/ u \in V(G)\}$. A node u is a fuzzy Steiner μ_k diametral node (or) peripheral node if $e_{\mu_k G}(u) = diam_{\mu_k G}(G)$. The fuzzy Steiner μ_k centre $C_{\mu_k}(G)$ of a connected fuzzy graph G is the sub graph induced by the nodes u of V(G) with $e_{\mu_k G}(u) = r_{\mu_k G}(G)$. The node u is called fuzzy Steiner μ_k central node (or) fuzzy Steiner μ_k eccentric node.

3. FUZZY STEINER μ DIMENSION

3.1: DEFINITION(fuzzy steiner μ representation)

Let G(V,σ,μ) be a connected fuzzy graph with n nodes. Let $S = \{u_1, u_2, \ldots, u_k\}$ be any subset of k nodes. Let v be any node in G. The fuzzy Steiner μ representation of the node 'v' concerned with S is stated as

$$s_\mu(v/S) = \{d_{\mu G}(S_1), d_{\mu G}(S_2), \ldots, d_{\mu G}(S_k), \ldots d_{\mu G}(S_{1,2}), d_{\mu G}(S_{1,3}), \ldots$$
$$d_{\mu G}(S_{1,2,3}), \ldots, d_{\mu G}(S_{1,2,3\ldots\ldots,k})\}$$

where $d_{\mu G}(S_{1,2,3\ldots\ldots,r})$ is the Steiner μ-distance of $S_{1,2,3\ldots\ldots,r} = \{v, u_1, u_2, \ldots u_r\}$. There are $kC_1 + kC_2 + kC_3 + \cdots + kC_k = 2^k - 1$ Steiner μ distances in each Steiner μ representation with respect to S.

3.2: DEFINITION(Steiner μ resolving set)

A subset of nodes S for a fuzzy graph G is termed to be a Steiner μ resolving set for G if $s_\mu(u/S) \neq s_\mu(v/S)$(i.e) every two nodes have different Steiner μ representation.

3.2: DEFINITION(fuzzy Steiner μ dimension)

A fuzzy Steiner μ resolving set of a fuzzy graph G with minimum fuzzy cardinality is said to be the Steiner μ basis for G and the summation of membership values of nodes of a Steiner μ basis of G is declared as the Steiner μ dimension of G denoted by $dim_\mu(G)$.

3.3: EXAMPLE

Let us find the fuzzy Steiner μ dimension of the following cocktail party fuzzy graph with 6 nodes.

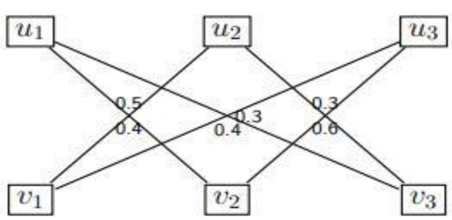

Fig: 1

Table.1: The Steiner μ representations of nodes of G with respect to singleton node sets

	$s_\mu(u_1/S_i)$	$s_\mu(u_2/S_i)$	$s_\mu(u_3/S_i)$	$s_\mu(v_1/S_i)$	$s_\mu(v_2/S_i)$	$s_\mu(v_3/S_i)$	Whether Steiner μ resolvin

							g set or not
$S_1 = \{u_1\}$	-	{4.167}	{5.83}	{6.17}	{2.5}	{2.5}	No
$S_2 = \{u_2\}$	{4.167 }	-	{5.33}	{2}	{6.67}	{1.67}	Yes
$S_3 = \{u_3\}$	{5.83}	{5.33 }	-	{3.33}	{3.33}	{7}	No
$S_4 = \{v_1\}$	{6.17}	{2}	{3.33}	-	{6.66}	{3.67}	Yes
$S_5 = \{v_2\}$	{2.5}	{6.67 }	{3.33}	{6.66}	-	{5}	Yes
$S_6 = \{v_3\}$	{2.5}	{1.67 }	{7}	{3.67}	{5}	-	Yes

From the above table, the minimal Steiner μ resolving sets are $S_2 = \{u_2\}$, $S_4 = \{v_1\}$, $S_5 = \{v_2\}$ and $S_6 = \{v_3\}$. Among these sets $S_4 = \{v_1\}$ is the minimum Steiner μ resolving set which is the Steiner μ basis of G. Therefore the Steiner μ dimension of G is $dim_\mu(G) = 0.5$.

4. EMBEDDING THEOREM ON STEINER μ_k CENTRE

Every connected fuzzy graph G with k nodes is the Steiner μ_k centre of some connected fuzzy graph with 'n' nodes where $k < n$.

Proof :

Let G be a connected fuzzy graph with k nodes. We shall construct a connected fuzzy graph G' with n nodes from the given graph G such that G is the steiner μ_k centre of G'.

Let $k = 2$. Here G is obviously the fuzzy graph which is complete with 2 nodes. Let the nodes be u_1, u_2. Here we shall construct G' by inserting two nodes x,y and the arcs u_1x, u_1y, u_2x, u_2y *and* xy. The weights of new arcs could be designated as follows. $\mu(u_1, x) = \mu(u_1, y) = \mu(u_2, x) = \mu(u_2, y) > \mu(u_1, u_2)$ and $\mu(x, y) < \mu(u_1, u_2)$. Now for the graph G' with $n = 2k$ nodes which is a complete fuzzy graph with 4 nodes, the 2-set $\{x, y\}$ has maximum steiner μ_2 distance and the 2-set $\{u_1, u_2\}$ has second highest steiner μ_2 distance. All the other 2-sets have same steiner μ_2 distance which is less than that of the set $\{u_1, u_2\}$. As a consequence, the steiner μ_2 eccentricities of the nodes x and y are similar and would be the maximum. The steiner μ_2 eccentricities of the nodes u_1 *and* u_2 are equal and minimum which is the steiner μ_2 radius G'.Thus the steiner μ_2 centre $C_{\mu_k}(G') = \langle u_1, u_2 \rangle = G$. On this account, G is the steiner μ_2 centre of the newly constructed graph G' which has 4 nodes.

Now assume that $k > 2$. G is a connected fuzzy graph with k nodes. Let the k nodes be indicated by $u_1, u_2, u_3 \ldots\ldots, u_k$ Now G' can be constructed by adding another k nodes

$v_1, v_2, v_3 \ldots\ldots, v_k$ to the graph G. The following arcs are added. Each v_i is joined with each u_j. Also each v_i is joined with v_{i+1} for $i = 1,2, \ldots., k-1$ and v_k is joined with v_1 so that $v_1, v_2, v_3 \ldots\ldots, v_k$ are the nodes of a fuzzy k-cycle. Now for each of the arcs $v_i v_{i+1}$ and for the arc $v_1 v_k$ identical arc weights are allocated which should be less than that of the least arc weight of the graph G. On the other hand to each of the arcs $v_i u_j$ same arc weights are assigned which is greater than that of the highest arc weight of the graph G.

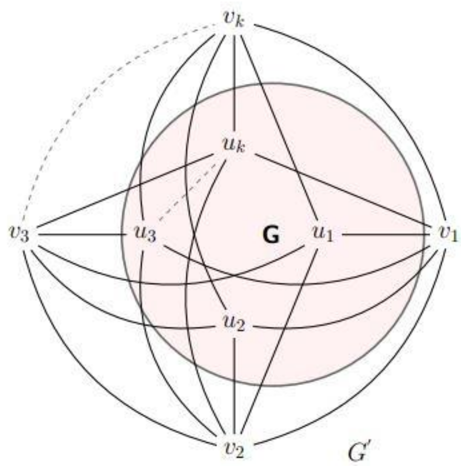

Fig: 2

The next process is to find the steiner μ_k centre of the new graph G'. Now examine the k-set of nodes $V_1 = \{v_1, v_2, v_3 \ldots\ldots, v_k\}$. The steiner trees of the aforementioned set of nodes holds only the newly annexed arcs. Since the arcs of the fuzzy cycle $v_1, v_2, v_3 \ldots\ldots, v_k$ have equal and least arc weights and in the computation of steiner μ_k distance the reciprocals of the arc weights are added, the set V_1 has maximum steiner μ_k distance.

Besides the steiner trees of the k-set of nodes $U_1 = \{u_1, u_2, u_3 \ldots\ldots, u_k\}$ are nothing but the spanning steiner trees of the graph G. For all the k-sets containing atleast one u_i and atleast one v_j the steiner trees contains only the newly added arcs with highest arc weights. Hence the steiner μ_k distances of these k-sets are equal and less than that of the set U_1. Therefore U_1 has the second maximum value for steiner μ_k distance. Thus the steiner μ_k eccentricities of the nodes of V_1 are same and is maximum. The steiner μ_k eccentricities of the nodes of U_1 are same and is minimum which is the steiner μ_k radius of the graph G'. So these nodes form the steiner μ_k centre of G'.

(i.e) $C_{\mu_k}(G') = \langle U_1 \rangle = G$

Accordingly, every connected fuzzy graph G possesses k nodes can be placed in a connected fuzzy graph G' with $n > k$ nodes such that G is the steiner μ_k centre of G'.

These connected fuzzy graphs which are constructed by this theorem are called steiner μ_k embedding graphs.

5. RESULTS AND DISCUSSION

This work introduces the definitions of Steiner μ representation, Steiner μ resolving set, Steiner μ basis and Steiner μ dimension. In addition the existence of Steiner μ_k embedding graphs have been proved which have application background in communication network problems.

6. CONCLUSION

This paper initiates the study of Steiner μ dimension in connected fuzzy graphs and its characteristic properties. Embedding theorem of Steiner μ_k centre has been established.

REFERENCE

[1] Benjier H.Arriola, "A NOTE ON SOME DOMINATION OF FUZZY GRAPHS", *Far East Journal of Applied Mathematics*,(2017) 113-123.

[2] K.R. Bhutani and A. Rosenfeld "STRONG ARCS IN FUZZY GRAPHS ", *Information Sciences,* 152(2003), 319-322.

[3] F. Buckley and F. Harary, "DISTANCE IN GRAPHS ", *Addison-Wesley Publishing Company,Inc.,* (1990).

[4] G. Chartrand and P. Zang, "INTRODUCTION TO GRAPH THEORY ". *Tata McGrawHill Edition,* (2006).

[5] F. Harary, "GRAPH THEORY ", *Addison-Wesley Publishing Company, Inc.,* (1969).

[6]Peter Dankelmann , Henda C. Swarta and Ortrud R. Oellermannb, ON THE AVERAGE STEINER DISTANCE OF GRAPHS WITH PRESCRIBED PROPERTIES, *Discrete Applied Mathematics* (1997), 91-103.

[7]A. Rosenfeld, FUZZY GRAPHS, *In Fuzzy Sets and their Application to Cognitive and Decision Processes, Academic Press,* (1975).

[8]M.S Sunitha and Sunil Mathew, FUZZY GRAPH THEORY: A SURVEY, ANNALS OF PURE AND APPLIED MATHEMATICS,(2013),92-110

[9] Yashwant Vaishnaw and Sanjay Sharma, SOME NEW APPROACHES OF FUZZY GRAPH,IOSR JOURNAL OF MATHEMATICS,(26016), 109-112

[10]Yaping Mao, STEINER DISTANCE IN GRAPHS—A SURVEY,(2017)