MORE FUNCTIONS RELATED TO NANO JD OPEN SETS

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ABSTRACT

We introduce a new sort of *Nano* JD continuous functions in *Nano* Topological spaces (M,τ(P)), namely *Nano* Strongly and Perfectly JD continuous functions in this article. Basic properties of *Nano* Strongly and Perfectly JD continuous functions are discussed. We establish their interconnection with other pre-exisiting concepts.

1. INTRODUCTION

Nano topology is one of the latest feathers in topology that applies to real life situations. Lellis Thivagar et al. [11] was the main brain behind developing the concept of *Nano* topology. The term *Nano* can be ascribed to anyunit of measure. Followed by which many mathematicians worked towards the generalisations of these sets in *Nano* topological spaces.

A new sort of *Nano* JD continuous functions namely *Nano* JD Perfectly and *Nano* JD Totally Continuous functions are introduced here.

2. PRELIMINARIES

This section establishes basic definitions and theorems related to the study of this article. Throughout this article $N_n$cl and $N_n$int represents *Nano* closure and *Nano* interior.

2.1 Definition

If (M,τ(P)) is a *Nano* Topological space with respect to P with J ⊆ M . Then J is claimed to be

1. *Nano* semi-open if J ⊆ $N_n$cl($N_n$int(J)) .
2. *Nano* pre-open if J ⊆ $N_n$int($N_n$cl(J)) .
3. *Nano* α − open if J ⊆ $N_n$int ($N_n$cl($N_n$int(J))).
4. *Nano* β − open if J ⊆ $N_n$cl ($N_n$int($N_n$cl(J))) .
5. *Nano* regular-open if J = $N_n$int($N_n$cl(J)).

2.2 Definition

A function k: (M,τ(P)) → (N,τ(Q)) is called *Nano* perfectly continuous if for each *Nano* open set in (N, τ(Q)) the inverse image is both *Nano* open and *Nano* closed in (M, τ(P)).

2.3 Definition

A function k:(M, τ(P)) → (N, τ(Q)) is called *Nano* open if the image of each *Nano* open set in (M, τ(P)) is *Nano* open in (N, τ(Q)).

2.4 Definition

A function k:(M, τ(P)) →(N, τ(Q)) is called *Nano* closed if the image of each *Nano* closed set in (M, τ(P)) is $N\acute{a}no$ closed in (N, τ(Q)) .

3.NANO PERFECTLY JD CONTINUOUS FUNCTION

Here we define $N\acute{a}no$ Perfectly JD-Continuous functions and studied their characterizations in $N\acute{a}no$ topological spaces.

3.1 Definition

A function k:(M, τ(P)) → (N, τ(Q)) is called a $N\acute{a}no$ perfectly JD continuous if for every $N\acute{a}no$ JD open set in (N,τ(Q)) the inverse image is both $N\acute{a}no$ open and $N\acute{a}no$ closed in (M,τ(P)).

3.2 Theorem

If a map k: (M, τ(P)) → (N, τ (Q)) is $N\acute{a}no$ perfectly JD continuous then it is $N\acute{a}no$ strongly JD continuous.

Proof :

Assume that k is $N\acute{a}no$ perfectly JD continuous. Let J be any $N\acute{a}no$ JD open set in (N, τ(Q)), Since k is $N\acute{a}no$ perfectly JD continuous, $k^{-1}(J)$ is both $N\acute{a}no$ open and $N\acute{a}no$ closed in (M, τ(P)). Therefore k is $N\acute{a}no$ strongly JD continuous.

3.3 Theorem

If a map k:(M,τ(P))→(N,τ(Q)) is $N\acute{a}no$ perfectly JD continuous then it is $N\acute{a}no$ perfectly continuous.

Proof:

Let J be an $N\acute{a}no$ open set in (N, τ(Q)), Since every $N\acute{a}no$ open set is $N\acute{a}no$ JD open set, J is a $N\acute{a}no$ JD open set in (N, τ(Q)). Since k is $N\acute{a}no$ perfectly JD continuous, $k^{-1}(J)$ is both $N\acute{a}no$ open and $N\acute{a}no$ closed in (M, τ(P)). Therefore k is $N\acute{a}no$ perfectly continuous.

3.4 Theorem

A map k:(M,τ(P)) → (N,τ(Q)) is $N\acute{a}no$ perfectly JD continuous if and only if k−1(J) is both $N\acute{a}no$ open and $N\acute{a}no$ closed in (M, τ(P)) for every $N\acute{a}no$ JD closed set J in (N, τ(Q)).

Proof:

Let J be any $N\acute{a}no$ JD closed set in (N, τ(Q)). Then J^c is $N\acute{a}no$ JD open in (N, τ(Q)). Since k is $N\acute{a}no$ perfectly JD continuous, $k^{-1}(J^c)$ is both Nano open and Nano closed in (M, τ(P)). But $k^{-1}(J)$ =
$M\backslash k^{-1}(J)$ and so $k^{-1}(J)$ is both $N\acute{a}no$ open and $N\acute{a}no$ closed in (M,τ(P)). But $k^{-1}(J^c) = M \backslash k^{-1}(J)$ and so $k^{-1}(J)$ is both $N\acute{a}no$ open and $N\acute{a}no$ closed in (M, τ(P)). Let J be any $N\acute{a}no$ JD open set in (N, τ(Q)). Then J^c is $N\acute{a}no$ JD closed in (N,τ(Q)).

By assumption $k-1(J^c)$ is both $N\acute{a}no$ open and $N\acute{a}no$ closed in (M, τ(P)). But $k^{-1}(J^c) = M\backslash k^{-1}(J)$ and so $k^{-1}(J)$ is both $N\acute{a}no$ open and $N\acute{a}no$ closed in (M,τ(P)). Therefore, k is $N\acute{a}no$ perfectly JD continuous.

3.5 Theorem

Let (M, τ(P)) be a discrete $N\acute{a}no$ topological space and (N,τ(Q)) be any $N\acute{a}no$ topological space. The statement below holds true for the map (M,τ(P)) →(N,τ(Q)),

(i) k is $N\acute{a}no$ strongly JD continuous.

(ii) k is $N\acute{a}no$ perfectly JD continuous.

Proof :

(i)⇒(ii) Let J be any *Nano* JD open set in (N, τ(Q)). By hypothesis, $k^{-1}(J)$ is *Nano* open in (M, τ(P)). Since τ(P)) is a discrete *Nano* space, $k^{-1}(J)$ is *Nano* closed in (N, τ (Q)). $k^{-1}(J)$ is both *Nano* open and *Nano* closed in (M, τ (P)). Hence, k is *Nano* perfectly JD continuous.

(ii)⇒ (i) Let J be any Nano JD open set in (N, τ(Q)). Then, $k^{-1}(J)$ is both *Nano* open and *Nano* closed in (M, τ (P)). Hence, k is *Nano* strongly JD continuous.

3.7 Theorem

Let k: (M, τ(P)) → (N, τ(Q)) and s:(N, τ(Q)) → (L,τ(O)) are *Nano* perfectly JD continuous functions then their composition s∘k : (M, τ (P)) → (L, τ(O)) is also *Nano* perfectly JD continuous.

Proof:

Let J be a *Nano* JD open set in (L, τ(O)). Since s is *Nano* perfectly JD continuous. We get that $s^{-1}(J)$ is *Nano* open and *Nano* closed in (N, τ(Q)). Then $s^{-1}(J)$ is *Nano* JD open in (N, τ(Q)). Since k is *Nano* perfectly JD continuous, $k^{-1}s^{-1}(J) = (s\circ k)^{-1}$ is both *Nano* open and *Nano* closed in
(M, τ(P)). Hence, s ∘ k is *Nano* perfectly JD continuous.

3.8 Theorem

Let k: (M, τ(P)) → (N, τ(Q)) and s: (N, τ(Q)) → (L, τ(O)) be any two maps. Then their composition is *Nano* JD continuous if s is *Nano* perfectly JD continuous and k is *Nano* continuous.

Proof :

Let J be any *Nano* JD open set in (L, τ(O)). Then, $s^{-1}(J)$ is both *Nano* open and *Nano* closed in (N, τ(Q)). since k is *Nano* continuous, $k^{-1}(s^{-1}(J)) = (s \circ k)^{-1}(J)$ is *Nano* open in (M,τ(P)). Hence s ∘ k is *Nano* strongly JD continuous.

3.8 Theorem

If a map k:(M, τ(P)) → (N, τ(Q)) is *Nano* perfectly JD continuous and a map s: (N, τ (Q)) → (L, τ(O)) is *Nano* strongly JD continuous then their composition is s ∘k : (M, τ(P)) → (L,τ(O)) is *Nano* perfectly JD continuous.

Proof :

Let J be any *Nano* JD open set in (L, τ(O)). Then $g^{-1}(J)$ is *Nano* open in (N, τ(Q)). Then $g^{-1}(J)$ is *Nano* JD open in (N, τ(Q)). By hypothesis, $k^{-1}(s^{-1}(J)) = (s \circ k)^{-1}(J)$ is both *Nano* open and *Nano* closed in (M, τ(P)). Therefore, s ∘ k is *Nano* perfectly JD continuous.

4.NANO TOTALLY JD CONTINUOUS FUNCTION

Here we define *Nano* Totally JD Continuous functions and studied their characterizations in *Nano* topological spaces.

4.1 Definition

A function k: (M, τ (P)) → (N, τ(Q)) is called *Nano* totally JD continuous if the inverse image of every *Nano* open set of (N, τ(Q)) is both *Nano* JD open and *Nano* JD closed set of
(M, τ(P)).

4.2 Example

Let M={g,h,i,j} with M/R= {{g}, {i},{h,j}} and P={g,h}.
Then the topology τ(P) = {M, φ, {g}, {g, h, j}, {h, j}},
NJDO(X) = {U, φ, {g}, {h}, {j}, {g, h}, {h, i}, {i, j}, {g, j},
{h, j}, {g, i}, {g, h, i}, {g, h, j}, {h, i, j}}, {g, i, j}}
NJDC(X) = {U, φ, {g}, {h}, {i}, {j}, {g, j}, {g, h}, {h, i},
{i, j}, {g, j}, {h, j}, {g, i}, {g, h, i}, {h, i, j}}, {g, i, j}}
Let N={g,h,i,j} with N/R'={{g,h,j},{i}} and Q={g}⊆ N.
Then τ (Q) = {N, φ, {g, h, j}}.
Define k:M→N as k(g)=g; k(h)=i; k(i)=j; k(j)=h.
Here k is Nano totally JD continuous.

4.3 Theorem

Let a mapping k: (M, τ(P)) → (N, τ(Q)), then every *Nańo* Totally JD continuous functions is Na´no JD continuous function.
Proof :
Let J be an open set in (N, τ(Q)). Since k is *Nańo* totally JD continuous functions,

k^{-1}(J) is both *Nańo* JD open and *Nańo* JD closed in (M, τ(P)). Therefore, k is *Nańo* JD continuous.

4.4 Remark

Reverse implications need not hold true as evidenced by the illustration below.

4.5 Example

Let M= {l,r,w,v} with M/R={{l},{v},{r,w}} and P= {l,w} .
Then the τ(P) = {M, φ, {l }, {w} , { r, w }, {l, r, w}} , Then N= {l,r,w,v} with N/R'={{l,v},{r},{w}} Q={l,w}. Then τ(Q) = {N, φ, {l,w,v},{l,v},{w}}.
Consider a function k:M→N as k(l)=r; k(r)=v; k(w)=w; k(v)=l.
Here k is *Nańo* JD continuous but not *Nańo* totally continuous.

4.6 Theorem

Let a mapping k:(M,τ(P)) →(N, τ(Q)), then every *Nańo* totally continuous function is *Nańo* totally JD continuous.
Proof :

Let J be a *Nańo* open set of (N, τ(Q)). Since k is *Nańo* totally continuous function,

k^{-1}(J) is both *Nańo* open and *Nańo* closed in (M, τ(P)). Since every *Nańo* open set is *Nańo* JD open and every *Nańo* closed set is *Nańo* JD closed. k^{-1}(J) is both *Nańo* JD open and *Nańo* JD closed in (M, τ(P)). Therefore, k is *Nańo* totally JD continuous.

4.7 Remark

Reverse implications need not hold true as evidenced by the illustration below.

4.8 Example

Let M={g,h,i,j} with M/R= {{g}, {i},{h,j}} and P={g,h}.
Then the topology τ(P) = {U, φ, {g }, {g, h, j} , {h, j} }
Let N= {l,r,w,v} with N/R'= {{l,r,v }, {w}} and Q= { l }. Then τ (Q) = {N, φ, {l, r, v}}.
Define k:M → N as k(g)=l; k(h)=w; k(i)=v; k(j)=r.

Here k is *Náno* totally JD continuous but not *Náno* totally continuous

4.9 Theorem

Consider a mapping h:(M, τ(P)) → (N, τ(Q)), then every *Náno* perfectly continuous function is *Náno* totally JD continuous function.

Proof:

Let h: (M, τ (P)) → (N, τ (Q)) be a *Náno* perfectly JD continuous map and J be any *Náno* open set in (N, τ (Q)). Then J is *Náno* JD open in (N, τ(Q)). Since h is *Náno* perfectly JD continuous, $h^{-1}(J)$ is both *Náno* open and *Náno* closed in (M, τ(P)), it implies $h^{-1}(J)$ is both *Náno* JD open and *Náno* JD closed in (M, τ(P)). Therefore, h is *Náno* totally JD continuous.

4.10 Remark

Reverse implications need not hold true as evidenced by the illustration below

4.11 Example

Let M={l,r,v,w,i} with M/R= {{l,r}, {v,w},{i}} and P={r,i}.

Then the *Náno* topology

τ(P) = {M, φ, {l, r, i}, {l, r}, {i}},

Then N={l,r,v,w,i} with

N/R'= {{l,r,v}, {i } ,{w}} and Q= {r,v,w} . Then τ(Q)= {N,φ, {l,r,v,w} , {w}, {l,r,v}}.

Define h:M→N as h(l)=r, h(r)=l, h(v)=v, h(w)=w; h(i)=i.

Here h is *Náno* Totally JD continuous but not *Náno* perfectly JD continuous.

4.12 Theorem

If h: M x N is a *Náno* totally JD continuous map, and M is *Náno* JD connected, then N is a *Náno* indiscrete space.

Proof:

Let's presume that N is not a *Náno* indiscrete space. And J be a non-empty *Náno* open subset of N. Since, h is *Náno* totally JD continuous map, then $h^{-1}(J)$ is a non-empty *Náno* JD clopen subset of M. Then M=$h^{-1}(J)$ U $(h^{-1}(J))^c$. Thus, M is a union of two non empty disjoint *Náno* JD open sets despite the fact that M is *Náno* JD connected this is a contradiction. Therefore, N must be an *Náno* indiscrete space.

4.13 Theorem

Let h:(M, τ (P)) → (N, τ(Q)) and s: (N,τ(Q)) → (L,τ(O)) be a map. Then

(i) If h is *Náno* JD irresolute and s is *Náno* totally JD continuous then s ∘ h is *Náno* totally JD continuous.

(ii) If h is *Náno* totally JD continuous and s is *Náno* continuous then s ∘ h is *Náno* totally JD continuous.

Proof:

(i) Let J be a *Náno* open set in (L, τ(O)). Since s is *Náno* totally JD continuous, $s^{-1}(J)$ is *Náno* JD clopen in (N, τ(Q)). Since h is *Náno* JD irresolute, $h^{-1}(s^{-1}(J))$ is *Náno* JD open and *Náno* JD closed in M. Since $(s \circ h)^{-1}(J) = h^{-1}(s^{-1}(J))$. Therefore, s ∘ h is *Náno* totally JD continuous.

(ii) Let J be an *Náno* open set in L. Since s is *Náno* continuous, $s^{-1}(J)$ is *Náno* open in N. Since, h is *Náno* totally JD continuous, $h^{-1}(s^{-1}(J))$ is *Náno* JD clopen in (M, τ(P)). Hence, s ∘ h is *Náno* totally JD continuous.

5. CONCLUSION

We have investigated the amazing attributes of *Nańo* Perfectly & Totally JD continuous functions in this work. Going one step ahead, we also establish their relationship with other current notions.

REFERENCE

[1] K. Bhuvaneshwari and K.M.Gnanapriya, "NANO GENERALISED CLOSED SETS IN NANO TOPOLOGICAL SPACES ", *International Journal of Scientific and Research Publication, 4(5),pp.1-3,2014.*

[2] R.T.Nachiyar and K.Bhuvaneshwari., "NANO GENERALISED α CLOSED SETS AND NANO α-GENERALISED IN NANO TOPOLOGICAL SPACES", *International Journal of Engineering Trends and Technology, 13(6), pp.257-260,2014.*

[3] I.L.Reilly and Vamanamurthy., "ON α- SETS IN TOPOLOGICAL SPACES", *Tamkang Journal of Mathematics, 16, pp.7-11,1985.*

[4] P.Anbarasi Rodrigo and K.Rajendra Suba., "FUNCTIONS RELATED TO β* CLOSED SETS IN TOPOLOGICAL SPACES", *International Journal of Mathematics Trends and Technology, Special Issue, March 2018.*

[5] M.L.Thivagar and C.Richard., "ON NANO FORMS OF WEAKLY OPEN SETS ", *International Journal of Mathematics and Statistics Invention, 1(1),pp 31-37,* 2013.