Mean 4- Square Sum E- Cordial Labeling of Some Simple Graphs

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ABSTRACT

In this paper , we explicates a new definition of Mean 4 – square sum E- cordial graph which is defined as follows: Let $G(V, E)$ be a simple graph and let $f: E(G) \to \{1,2,3,4\}$ be a mapping with the induced labeling $f^*: V(G) \to \{0,1\}$ for each vertex u assign the label 1 if $\left\lfloor\frac{\sum f(uv)^2}{\deg(u)}\right\rfloor$ is odd assign the label 0 if $\left\lfloor\frac{\sum f(uv)^2}{\deg(u)}\right\rfloor$ is even. The mapping f is called a mean 4- square sum E - cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ where $v_f(0)$ and $v_f(1)$ is the number of vertices labeled with 0 and labeled by 1 respectively. A graph which admits mean 4- square sum E - cordial labeling is called mean 4- square sum E - cordial graph.

1. Introduction

We begin with finite, connected and undirected graph without loops and multiple edges. Throughout this paper $|V(G)|$ and $|E(G)|$ respectively denote the number of vertices and edges in G. A graph labeling is an assignment of integer of the vertices of edges or both subject to certain conditions. If the domain of the mapping is the set of vertices or edges then the labeling is called a vertex labeling and edge labeling. A mapping f is called binary vertex labeling of G and $f(u)$ is called the label of vertex of G under f. For an edge $e = uv$, the induced edge labeling $f^*: E(G) \to \{0,1\}$ is given by $f^*(e = uv) = |f(u) - f(v)|$ then $v_f(i)$ is the number of vertices of G having label 1 under f and $e_f(i)$ is the number of edges of G having label 1 under f^* for $i = 0,1$. A binary vertex labeling of a graph G is called cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is a called coordinal if it admits coordinal labeling . Let G be a graph with vertex set $V(G)$ and edge set $E(G)$ and $f: E(G) \to \{0,1\}$ define f^* on V(G) by $f^*(v) = f^*(u) = \{\sum(f(uv))/uv \in E(G)\}(\,mod\ 2)$. The function f is called $E -$ coordinal labeling of G if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph is called E- coordinal if it admits E- Cordial labeling. In this paper we introduced a new definition of mean 4- square sum E-cordial graph which is defined .

Definition 1.1.

Let $G(V, E)$ be a simple graph and let $f: E(G) \to \{1,2,3,4\}$ be a mapping with the induced labeling $f^*: V(G) \to \{0,1\}$ for each vertex u assign the label 1 if $\left\lfloor\frac{\sum f(uv)^2}{\deg(u)}\right\rfloor$is odd assign the label 0 if $\left\lfloor\frac{\sum f(uv)^2}{\deg(u)}\right\rfloor$ is even. The mapping f is called a mean 4- square sum E - cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ where $v_f(0)$ and $v_f(1)$ is the number of vertices labeled with 0 and labeled by 1 respectively. A graph which admits mean 4- square sum E - cordial labeling is called mean 4- square sum E - cordial graph.

Example 1.2

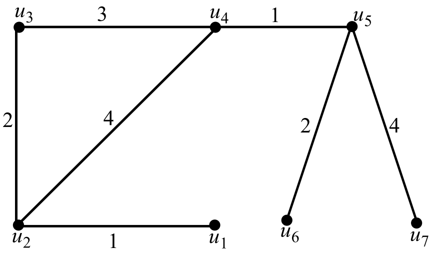

Fig. 1.1 Example of mean 4-square sum E - cordial labeling

From the above graph

$f^*(u_1) = 1, f^*(u_2) = 1, f^*(u_3) = 0, f^*(u_4) = 0,$

$f^*(u_5) = 1, f^*(u_6) = 0, f^*(u_7) = 0$

Hence $v_f(0) = 4, v_f(1) = 3$ this implies $|v_f(0) - v_f(1)| \leq 1$

Hence G is a mean 4- square sum E - cordial graph.

2. Mean 4- **Square Sum E - cordial Labeling For Some Standard Graph**

In this section we discuss some general graphs are mean 4- square sum E - cordial graph.

Observation 2.1.

The complete graphs are mean 4- square sum E - cordial graph if $n \leq 4$.

Example 2.2.

Consider K_4

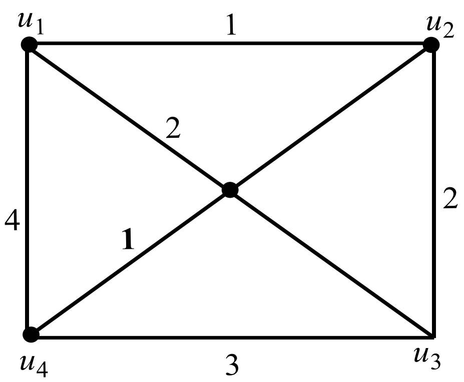

Fig.2.1.Mean 4- square sum E - cordial labeling of K_4

In the above graph $f^*(u_1) = 1, f^*(u_2) = 0, f^*(u_3) = 1, f^*(u_4) = 0,$
and $v_f(0) = 2, v_f(1) = 2$ this implies $|v_f(0) - v_f(1)| \leq 1$

Hence $K_4$is mean 4- square sum E - cordial labeling

The complete graphs are mean 4- square sum E - cordial graph if $n \leq 4$.

Theorem 2.3. Star graph $K_{1,n}$ is mean 4- square sum E - cordial graph if n is even

Proof: Let $V(K_{1,n}) = \{u, u_1, u_2, \dots, u_n\}$ be the vertices and $E(K_{1,n}) = \{uu_1, uu_2, uu_3, \dots, uu_n\}$ be the edges.

To define the labeling function $f: E(G) \to \{1,2,3,4\}$ is defined as follows

$$f(uu_i) = \begin{cases} 1 \ if \ i \equiv 1 \ (mod\ 4) \\ 2 \ if \ i \equiv 2 \ (mod\ 4) \\ 3 \ if \ i \equiv 3 \ (mod\ 4) \\ 4 \ if \ i \equiv 0 \ (mod\ 4) \end{cases}$$

Define f^* on V(G) by $f^*(u) = \begin{cases} 0 \ if \ \left\lfloor \frac{\sum f(uv)^2}{\deg(u)} \right\rfloor \ is\ even \\ 1 \ if \ \left\lfloor \frac{\sum f(uv)^2}{\deg(u)} \right\rfloor \ is\ odd \end{cases}$

The labeling defined as above satisfies $|v_f(0) - v_f(1)| \leq 1$

Hence star graph admits mean 4- square sum E - cordial labeling.

That is star graph is a mean 4- square sum E - cordial graph.

Example 2.4.

Consider $K_{1,6}$

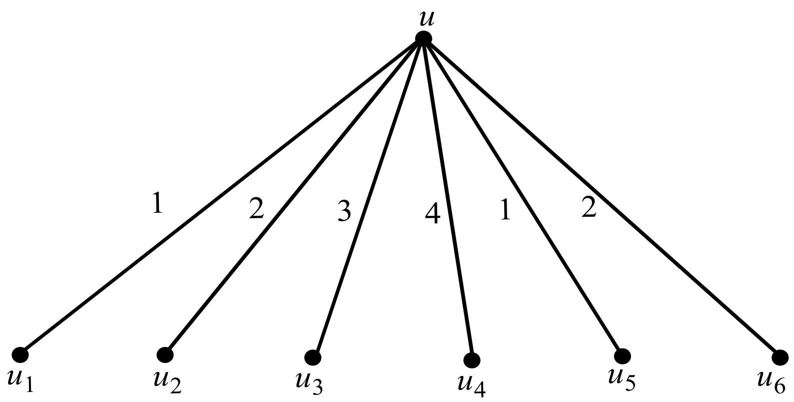

Fig.2.2.Mean 4- square sum E - cordial labeling of $K_{1,6}$

In the above graph

$f^*(u_1) = 1, f^*(u_2) = 0, f^*(u_3) = 1, f^*(u_4) = 0, f^*(u_5) = 1, f^*(u_6) = 0 f^*(u) = 1,$

Hence $v_f(0) = 3, v_f(1) = 4$ this implies $|v_f(0) - v_f(1)| \leq 1$

Hence $K_{1,6}$ is a mean 4- square sum E - cordial graph

Theorem 2.5.

Path graph P_n is mean 4- square sum E - cordial graph if n is odd.

Proof: Let $V(P_n) = \{u_1, u_2, \dots, u_n\}$ be the vertices and

$E(P_n) = \{u_1u_2, u_2u_3, u_3u_4, \dots, u_{n-1}u_n\}$ be the edges.

Suppose n is odd

To define the labeling function $f: E(G) \to \{1,2,3,4\}$ is defined as follows

$$f(u_iu_{i+1}) = \begin{cases} 1 \ if \ i \equiv 1 \ (mod\ 4) \\ 2 \ if \ i \equiv 2 \ (mod\ 4) \\ 3 \ if \ i \equiv 3 \ (mod\ 4) \\ 4 \ if \ i \equiv 0 \ (mod\ 4) \end{cases}$$

Define f^* on V(G) by $f^*(u) = \begin{cases} 0 \ \ if \ \ \left\lfloor \frac{\sum f(uv)^2}{\deg(u)} \right\rfloor \ is \ even \\ 1 \ \ if \ \ \left\lfloor \frac{\sum f(uv)^2}{\deg(u)} \right\rfloor \ is \ odd \end{cases}$

The labeling defined as above satisfies $\left|v_f(0) - v_f(1)\right| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$.
Hence path admits mean 4- square sum E - cordial labeling.
That is path is a mean 4- square sum E - cordial graph.

Theorem 2.6.

Bistar $B_{n,n}$ is mean 4- square sum E - cordial graph if n is odd

Proof: Let $V(B_{n,n}) = \{u, v, u_i, v_i, 1 \leq i \leq n\}$ be the vertices and
$E(B_{n,n}) = \{uv, uu_i, vv_i 1 \leq i \leq n\}$ be the edges.
Define the labeling function $f: E(G) \to \{1,2,3,4\}$ is defined as follows
For $1 \leq i \leq n$
$f(uv) = 3$

$f(uu_i) = \begin{cases} 1 \ if \ i = 1,3(mod 4) \\ 3 \ if \ i = 0,2(mod 4) \end{cases}$

$f(vv_i) = \begin{cases} 2 \ if \ i = 1,3(mod 4) \\ 4 \ if \ i = 0,2(mod 4) \end{cases}$

Define f^* on V(G) by $f^*(u) = \begin{cases} 0 \ \ if \ \ \left\lfloor \frac{\sum f(uv)^2}{\deg(u)} \right\rfloor \ is \ even \\ 1 \ \ if \ \ \left\lfloor \frac{\sum f(uv)^2}{\deg(u)} \right\rfloor \ is \ odd \end{cases}$

The labeling defined as above satisfies $|v_f(0) - v_f(1)| \leq 1$and $|e_f(i) - e_f(j)| \leq 1$.
Hence Bistar $B_{n,n}$admits mean 4- square sum E - cordial labeling.
That is Bistar $B_{n,n}$is a mean 4- square sum E - cordial graph.

Example 2.7. Consider $B_{5,5}$

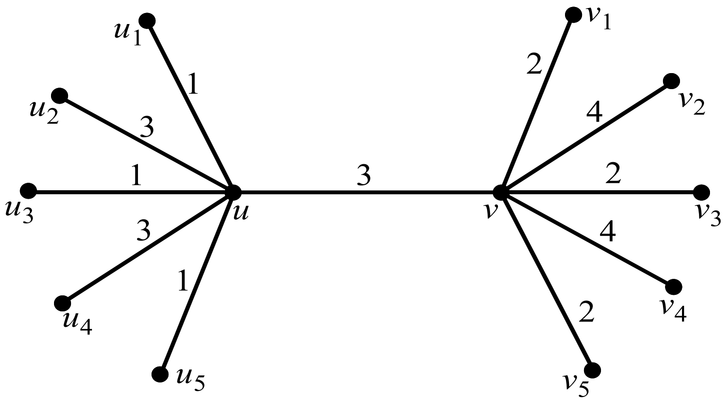

Fig.2.3.Mean 4- square sum E - cordial labeling of $B_{5,5}$

In the above graph

$f^*(u) = 1, f^*(u_1) = 1, f^*(u_2) = 1, f^*(u_3) = 1, f^*(u_4) = 1, f^*(u_5) = 1$
$f^*(v) = 0, f^*(v_1) = 0, f^*(v_2) = 0, f^*(v_3) = 0, f^*(v_4) = 0, f^*(v_5) = 0$

Hence $v_f(0) = 6, v_f(1) = 6$ this implies $|v_f(0) - v_f(1)| \leq 1$
Hence $B_{5,5}$ is a mean 4- square sum E - cordial graph

Theorem 2.8.

Coconut tree $CT_{n,n}$ is mean 4- square sum E - cordial graph

Proof : Let $V(CT_{n,n}) = \{u_1, u_2, u_3, \ldots, u_n, v_1, v_2, v_3, \ldots, v_n\}$ be the vertices and

$E\left(CT_{n,n}\right) = \{u_1u_2, u_2u_3, u_3u_4, \dots, u_{n-1}u_n, u_nv_1, u_nv_2, u_nv_3, \dots, u_nv_n\}$ be the edges. Here u_n is support vertex and $v_1, v_2, v_3, \dots, v_n$ are the pendent vertices.
Define the labeling function $f: E(G) \to \{1,2,3,4\}$ is defined as follows

$$f(u_iu_{i+1}) = \begin{cases} 2 & if\ i = 1,3\ (mod\ 4) \\ 4 & if\ i = 0,2\ (mod\ 4) \end{cases}$$

$$f(u_nv_i) = \begin{cases} 1 & if\ i = 1,3\ (mod\ 4) \\ 3 & if\ i = 0,2\ (mod\ 4) \end{cases}$$

Define f^* on V(G) by $f^*(u) = \begin{cases} 0 & if\ \left\lfloor \frac{\sum f(uv)^2}{\deg(u)} \right\rfloor\ is\ even \\ 1 & if\ \left\lfloor \frac{\sum f(uv)^2}{\deg(u)} \right\rfloor\ is\ odd \end{cases}$

The labeling defined as above satisfies $|v_f(0) - v_f(1)| \leq 1$and $|e_f(i) - e_f(j)| \leq 1$.
Hence Coconut tree $CT_{n,n}$ admits mean 4-square sum E - cordial labeling.
That is Coconut tree $CT_{n,n}$ is a mean 4-square sum E - cordial graph

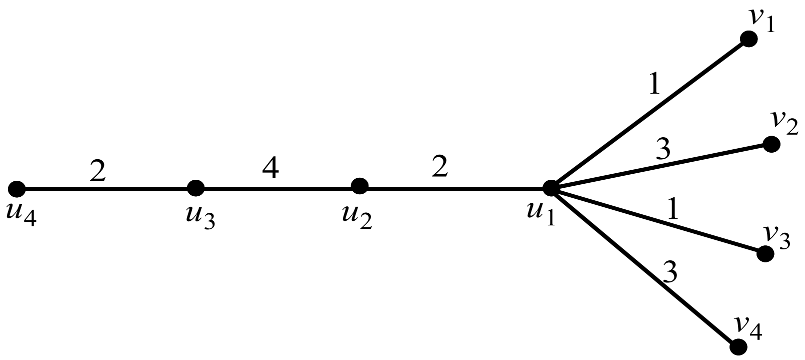

Fig.2.4.Mean 4- square sum E - cordial labeling of $CT_{4,4}$

Theorem 2.9. Crown graph $C_n \odot K_2$ is mean 4- square sum E - cordial graph if $n = 0(mod\ 4)$

Proof: Let $V(C_n \odot K_2) = \{u_1, u_2, u_3, \dots, u_n, u_1', u_2', u_3', \dots, u_n'\}$ be the vertices and

$E(C_n \odot K_2) = \{u_1u_2, u_2u_3, u_3u_4, \dots, u_nu_1, u_1u_1', u_2u_2', u_3u_3', \dots, u_nu_n'\}$ be the edges.
To define the labeling function $f: E(G) \to \{1,2,3,4\}$ is defined as follows
if $n = 0(mod\ 4)$
For $1 \leq i \leq n-1$

$$f(u_iu_{i+1}) = \begin{cases} 1\ if\ i \equiv 1\ (mod\ 4) \\ 3\ if\ i \equiv 2\ (mod\ 4) \\ 2\ if\ i \equiv 3\ (mod\ 4) \\ 4\ if\ i \equiv 0\ (mod\ 4) \end{cases}$$

For $1 \leq i \leq n$

$$f\left(u_iu_i'\right) = \begin{cases} 1\ if\ i \equiv 1\ (mod\ 4) \\ 3\ if\ i \equiv 2\ (mod\ 4) \\ 2\ if\ i \equiv 3\ (mod\ 4) \\ 4\ if\ i \equiv 0\ (mod\ 4) \end{cases}$$

Define f^* on $V(G)$ by $f^*(u) = \begin{cases} 0 & if\ \left\lfloor \frac{\sum f(uv)^2}{\deg(u)} \right\rfloor\ is\ even \\ 1 & if\ \left\lfloor \frac{\sum f(uv)^2}{\deg(u)} \right\rfloor\ is\ odd \end{cases}$

The labeling defined as above satisfies $\left|v_f(0) - v_f(1)\right| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$.

Hence Crown graph $C_n \odot K_2$ admits mean 4- square sum E -cordial labeling.
That is Crown graph $C_n \odot K_2$ is a mean 4- square sum E- cordial graph.

REFERENCE

1. Ponraj.R., Sivakumar.M and Sundram.M, Mean Cordial Labeling of Graphs, open Journal of Discrete mathematics, 2012,2,145-148.
2. Somasundaram.S and Ponraj.R, Mean Labeling of Graphs, National Academy Science, Letter, 26,2013, 210, 2013.
3. William.A, Rajasingh.I and Roy.S, Mean Cordial Labeling of Graphs, Journals, J.comp of Math.Sc., vol.4, No:4, 2013, 274-281.
4. Dhanalakshmi.S and Parvathi, Mean Square Cordial Labeling related to cyclic graphs, International Journal of Engineering and Technology.
5. Gayathiri.B and Sivakumar.C, Some Mean Cordial Lableing of Graphs, International Journal of Scientific Research and Review, vol:8, No.7, 2019, 263-285.