On Geometry and Psychology

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Abstract:
The aim of the paper is to brief out an (α, β)-metric and its applications to Psychology. Further, we provide some interesting results on geometry and psychology.
Key words: Randers metric, Finsler Geometry, Curvature.
AMS subject Classification (2010): 53B40, 53C60

1. Introduction and Preliminaries

Homogeneous spaces (in particular, Lie groups) equipped with invariant metrics have many applications in Physics. The study of homogeneous spaces (Lie groups) with invariant Riemannian metrics has been a very interesting field in recent decades ([1],[5],[8]).

The concept of (α, β)-metric was introduced by M. Matsumoto in 1972 and studiedby many authors like ([8],[10]).

Definition: The Finsler space $F^n = (M^n, L)$is said to have an (α, β) metric, if L is a positively homogeneous function of degree one in two arguments $\alpha = \sqrt{a_{ij}(x)y^i y^j}$and $\beta = b_i(x)y^i$, where α is a Riemannian metric and β, a differential 1-form.

2. Geometry and Psychology

A Randers metric is a natural and important Finsler metric which is defined as the sum of a Riemannian metric and a 1-form.
In [2], X. Cheng and Z. Shenfound Randers metrics are natural and important Finsler metrics which are defined asthe sum of a Riemann metric and a 1-form. They were derived from the research onthe general relativity and have been widely applied in many areas of natural science,including biology, physics and psychology, etc. In particular, Randers metrics can benaturally deduced as the solution of Zermelo navigation problem. Randers metricsare computable. Thus people can do in-depth computation of various geometricquantities, hence can understand the geometric properties of such metrics. Moreimportantly, Randers metrics have very rich non-Riemann curvature properties. Thestudy of Randers metrics will lead to a better understanding on Finsler metrics.

In [6], Ningwei Cui calls the flag curvature of a Finsler surface as the Gaussian curvature in Finsler geometry. In this paper, he characterizes the surfaces of constant Gaussian curvature (CGC) in the Randers 3-manifold. Then he gives a classification of the orientable closed CGC surfaces in two Randers space forms, which are the non-Euclidean Minkowski–Randers 3-space (K = 0) and the Bao–Shen sphere (K = 1). Similar to the Riemannian setting, it is an important problem tostudy the CGC Finsler surfaces in Randers 3-manifolds. However, it is in general difficult to give a classification even for theCGC surfaces in Riemannian 3-manifolds. In this paper, he deals with the closed (i.e. compact without boundary) orientable surfaces of constant Gaussian curvature in Randers 3-manifolds.

In [11], William C. Hoffman explains the generalization of systems theory in terms ofmodem differential geometry that takes proper account of nonlinear and “global” phenomena.As the geometry of systems also has application toperceptual psychology, for the invariances embodied in the perceptual constancies areactually spatiotemporal invariants of certain Lie transformation groups that occur inEuclidean and non-Euclidean geometry.

In [4], Leopold Verstraelen discusses on the geometry of the human kind.

In [12],William L. Abler explains certain advantages inherent in the absolutely basic nature of the principles that underlie the property of mind, as understood under the geometric/algebraic theory. The geometric/algebraic system is precise enough and basic enough to place limits on the kind of underlying mechanism that might generate it in the brain.

In [13], William C. Hoffmanexplains the concept of structuralstability which plays an important role in analyzing the connectivity of the neural network as "brains are as different as faces".
Also uses the Lukasiewicz's theory of parentheses to obtain a graph-theoretic of the Jacobi identity, which then serves to explain the branching of neuronal processes.

In[9], Stefan Haesen, Ana Irina Nistor, and Leopold Verstraelen illustrate the curious phenomenon by drawing the outline of a little *Nautilus* shell within a big one. We know, or we may see at once, that they areof precisely the same shape; so that, if we look at the little shell through a magnifyingglass, it becomes identical with the big one. But we know, on the other hand, thatthe little *Nautilus* shell grows into the big one, not by growth or magnification in all
parts and directions, as when a boy grows into a man, but by growing *at one endonly*.

In [7], Olivia López-González, Hérica Sánchez-Larios, ServioGuillén-Burguetecharacterize geodesics in the small in multidimensional psychological spaces. They highlight the following:
“The distance measure in the small does not fulfill the triangle inequality”,
“The geodesics in the small can be obtained from sets of tangent vectors with sum **v**” and
“There exists one and only one F-face for each direction **v**”.

In [3], Hoffman, W. C. explains Visual illusions of angle in terms of a calculus of visual constancies laid down earlier as misapplication of constancy. The rule is as follows: identify the curves appearingin the visual illusion as orbits of the appropriate visual constancy (or constancies). Keep the lie derivative that corresponds to the constancy whose orbit(s) appear distorted inthe illusion, but replace the other by the lie derivative orthogonal to it.

Form a linear combination of the resulting 2 lie derivatives, weighting the 1 that is kept the morestrongly. This linear combination will generate the distorted portion of the illusion.

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