GROUP S_3 CORDIAL SUM DIVISOR LABELING OF SOME DERIVED GRAPHS

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Abstract. Let $G = (V(G), E(G))$ be a graph and let $h : V(G) \rightarrow S_3$ be a function. For each edge uv assign the label 1 if 2 divides $(O(h(u))+O(h(v)))$ and 0 otherwise. The function h is called a Group S_3 cordial sum divisor labeling of G if $|v_h(i) - v_h(j)| \leq 1$ and $|e_h(1) - e_h(0)| \leq 1$ where $v_h(k)$ denote the number of vertices labeled with k, $k \in S_3$ and $e_h(1)$ and $e_h(0)$ denote the number of edges labeled with 1 and 0 respectively. A graph G which admits a Group S_3 cordial sum divisor labeling is called a Group S_3 cordial sum divisor graph. In this paper, we investigate the Group S_3 cordial sum divisor labeling of some derived graphs.

1. Introduction

For graph theoretical terminology, we refer to [2]. All graphs considered hereare simple, finite, connected and undirected. There are several types of labeling anda detailed survey of graph labeling can found in [3]. Cordial labeling is a weakerversion of graceful labeling and Harmonious labeling introduced by I.Cahit in [1]. Sum divisor cordial labeling was due to Lourdusamy and Patrick[5].The concept ofGroup S_3 cordial prime labeling was due to Kala and Chandra [4].

Motivated by these concepts, we introduce the concept of Group S_3 cordial sum divisor labeling. In this paper we investigate the Group S_3 cordial sum divisor labeling of some derived graphs.

Definition 1.1. Let A be a group. The order a ∈ A is the least positive integer n such that $a^n = e$.We denote the order of a by o(a).

Definition 1.2. Consider the Symmetric group S_3.Let the element of S_3 be e, a, b, c, d, f where

$$e = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$
$$a = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$$
$$b = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$

$$c = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$
$$d = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$$
$$f = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

we have $O(e) = 1, O(a) = O(b) = O(c) = 2$, and $O(d) = O(f) = 3$.

Definition 1.3. A Helm H_n is constructed from a wheel W_n by adding n vertices of degree one adjacent to each terminal vertex.

Definition 1.4. A Flower graph Fl_n is constructed from a helm H_n by joining each vertex of degree one to the center.

Definition 1.5. The Closed Helm CH_n is the graph obtained from helm H_n by joining each pendant vertex form a cycle.

Definition 1.6. For a simple connected graph G, the square of graph G is denoted by G^2 and defined as the graph with the same vertex set as of G and two verticesare adjacent in G^2 if they are at a distance 1 or 2 apart in G.

Definition 1.7. The graph $H_n \odot k_1$ is obtained by adding a pendant edge to each vertex of an H-graph H_n.

Definition 1.8. Globe graph is defined as the two isolated vertex are joined by n paths of length 2.It is denoted by Gl(n).

Definition 1.9. Let G be a graph and let $G_1 = G_2 = \cdots G_n$ where $n \geq 2$, then the graph obtained by adding an edge from each G_i to G_{i+1} $(1 \leq i \leq n - 1)$ is called the path union of G.

2. MAIN RESULTS

Theorem 2.1. The Flower graph Fl_n is a Group S_3 cordial sum divisor graph.

Proof. Let $G = Fl_n$. Let $V(G) = \{v, v_i, u_i : 1 \leq i \leq n\}$ and $E(G) = \{vv_i, v_iu_i, vu_i : 1 \leq i \leq n; v_nv_1; v_iv_{i+1} : 1 \leq i \leq n-1\}$. Then G is of order 2n + 1 and size 4n.
Define $h : V(G) \rightarrow S_3$ as follows

$$h(v_i) = \begin{cases} a \ \ if\ i \equiv 1 \ (mod\ 3)\ and\ 1 \leq i \leq n \\ b\ if\ i \equiv 2\ (mod\ 3)\ and\ 1 \leq i \leq n \\ c\ if\ i \equiv 0\ (mod\ 3)\ and\ 1 \leq i \leq n \end{cases}$$

$$h(u_i) = \begin{cases} d\ \ if\ i \equiv 1\ (mod\ 3)\ and\ 1 \leq i \leq n \\ e\ if\ i \equiv 2\ (mod\ 3)\ and\ 1 \leq i \leq n \\ f\ if\ i \equiv 0\ (mod\ 3)\ and\ 1 \leq i \leq n \end{cases}$$

TabLE 1

Nature of n	$v_h(a)$	$v_h(b)$	$v_h(c)$	$v_h(d)$	$v_h(e)$	$v_h(f)$	$e_h(0)$	$e_h(1)$
$6k+1 (k \geq 1)$	$2k+1$	$2k$	$2k+1$	$2k+1$	$2k$	$2k$	$12k+2$	$12k+2$
$6k+2 (k \geq 1)$	$2k+1$	$2k+1$	$2k+1$	$2k+1$	$2k+1$	$2k$	$12k+4$	$12k+4$
$6k+3 (k \geq 0)$	$2k+1$	$2k+1$	$2k+1$	$2k+1$	$2k+1$	$2k+1$	$12k+6$	$12k+6$
$6k+4 (k \geq 0)$	$2k+2$	$2k+1$	$2k+1$	$2k+2$	$2k+1$	$2k+1$	$12k+8$	$12k+8$
$6k+5 (k \geq 0)$	$2k+2$	$2k+2$	$2k+1$	$2k+2$	$2k+2$	$2k+1$	$12k+10$	$12k+10$
$6k (k \geq 1)$	$2k$	$2k$	$2k+1$	$2k$	$2k$	$2k$	$12k$	$12k$

TabLE 2

Nature of n	$v_h(a)$	$v_h(b)$	$v_h(c)$	$v_h(d)$	$v_h(f)$	$v_h(e)$	$e_h(0)$	$e_h(1)$
$3k+1 (k \geq 1)$	$k+1$	k	k	$k+1$	k	$k+1$	$6k+2$	$6k+2$
$3k+2 (k \geq 1)$	$k+1$	$k+1$	k	$k+1$	$k+1$	$k+1$	$6k+4$	$6k+4$
$3k (k \geq 1)$	k	k	k	k	k	$k+1$	$6k$	$6k$

From **TABLE** 1,we can easily verify that $|v_h(\mathbf{i}) - v_h(\mathbf{j})| \leq 1$ for every $\mathbf{i}, \mathbf{j} \in S_3$ and $|e_h(1) - e_h(0)| \leq 1$ Therefore,the Flower graph $\mathbf{Fl_n}$ admits a Group S_3 cordial sum divisor labeling. Hence the Flower graph $\mathbf{Fl_n}$ is a group S_3 cordial sum divisor graph.

Theorem 2.2. The Closed Helm graph CH_n is a Group S_3 cordial sum divisor graph.

Proof. Let $G = CH_n$. Let $V(G) = \{v, v_1, v_2, v_3, \cdots v_n, u_1, u_2, u_3, \cdots u_n\}$ and
$E(G) = \{vv_i, u_iv_i : 1 \leq i \leq n\} \cup \{v_iv_{i+1}, u_iu_{i+1} : 1 \leq i \leq n-1\} \cup \{v_1v_n, u_1u_n\}$
Then G is of order $2n + 1$ and size $4n$. Define $h : V(G) \to S_3$ as follows
$h(u) = e_{\square}$

$$h(v\mathrm{i}) = \begin{cases} a \;\; if \;\; i \equiv 1 \;(mod\; 3) \; and \; 1 \; \leq i \; \leq n \\ b \; if \;\; i \equiv 2 \;(mod\; 3) \; and \; 1 \; \leq i \; \leq n \\ c \; if \;\; i \equiv 0 \;(mod\; 3) \; and \; 1 \; \leq i \; \leq n \end{cases}$$

$$h(u_i) = \begin{cases} d \ if \ i \equiv 1 \ (mod \ 3) \ and \ 1 \le i \le n \\ f \ if \ i \equiv 2 \ (mod \ 3) \ and \ 1 \le i \le n \\ e \ if \ i \equiv 0 \ (mod \ 3) \ and \ 1 \le i \le n \end{cases}$$

From TABLE 2,we can easily verify that $|v_h(\mathbf{i}) - v_h(\mathbf{j})| \le 1$ for every $\mathbf{i}, \mathbf{j} \in S_3$ and $|e_h(1) - e_h(0)| \le 1$. Therefore, the Closed Helm graph CH_n admits a Group S_3 cordial sum divisor labeling. Hence the Closed Helm graph CH_n is a group S_3 cordial sum divisor graph.

Theorem 2.3. $P_n{}^2$, $n \ge 3$ is a Group S_3 cordial sum divisor graph.

Proof. Let $G = P_n{}^2$. Let $v_1, v_2, v_3, \cdots v_n$ be the vertices of the Path P_n.
Let $V(G) = \{v_1, v_2, v_3, \cdots, v_n\}$ be the vertices of the path $P_n{}^2$ and $E(G) = \{v_iv_{i+1} : 1 \le i \le n - 1\} \cup \{v_iv_{i+2} : 1 \le i \le n - 2\}$Then $P_n{}^2$ is of order n and size $2n - 3$. Define $h : V(G) \to S_3$ as follows

$$h(v_i) = \begin{cases} d \ if \ i \equiv 1 \ (mod \ 6) \ and \ 1 \le i \le n \\ a \ if \ i \equiv 2 \ (mod \ 6) \ and \ 1 \le i \le n \\ e \ if \ i \equiv 3 \ (mod \ 6) \ and \ 1 \le i \le n \end{cases}$$

$$h(v_i) = \begin{cases} b \ if \ i \equiv 4 \ (mod \ 6) \ and \ 1 \le i \le n \\ f \ if \ i \equiv 5 \ (mod \ 6) \ and \ 1 \le i \le n \\ c \ if \ i \equiv 0 \ (mod \ 6) \ and \ 1 \le i \le n \end{cases}$$

TABLE 3

Nature of n	$v_h(d)$	$v_h(a)$	$v_h(e)$	$v_h(b)$	$v_h(f)$	$v_h(c)$	$e_h(0)$	$e_h(1)$
$6k + 1 (k \ge 1)$	$k + 1$	k	k	k	k	k	$6k$	$6k - 1$
$6k + 2 (k \ge 1)$	$k + 1$	$k + 1$	k	k	k	k	$6k + 1$	$6k$
$6k + 3 (k \ge 0)$	$k + 1$	$k + 1$	$k + 1$	k	k	k	$6k + 2$	$6k + 1$
$6k + 4 (k \ge 0)$	$k + 1$	$k + 1$	$k + 1$	$k + 1$	k	k	$6k + 3$	$6k + 2$
$6k + 5 (k \ge 0)$	$k + 1$	$k + 1$	$k + 1$	$k + 1$	$k + 1$	k	$6k + 4$	$6k + 3$
$6k (k \ge 1)$	k	k	k	k	k	k	$6k - 1$	$6k - 2$

TABLE 4

Nature of n	$v_h(a)$	$v_h(d)$	$v_h(c)$	$v_h(e)$	$v_h(b)$	$v_h(f)$	$e_h(0)$	$e_h(1)$
$6k + 2 (k \ge$	$k +$	$k +$	k	k	k	k	$6k +$	$6k +$

1)	1	1					2	2
$6k+4(k \geq 0)$	$k+1$	$k+1$	$k+1$	$k+1$	k	k	$6k+4$	$6k+4$
$6k(k \geq 1)$	k	k	k	k	k	k	$6k$	$6k$

From **TABLE** 3,we can easily verify that $|v_h(\mathbf{i}) - v_h(\mathbf{j})| \leq 1$ for every $\mathbf{i}, \mathbf{j} \in S_3$ and $|e_h(1) - e_h(0)| \leq 1$ Therefore, $P\ ^2$,n ≥ 3 admits a Group S_3 cordial sum divisor labeling. Hence $P_n{}^2 \geq 3$ is a group S_3 cordial sum divisor graph.

Theorem 2.4. The Square graph C_n^2, n is even admits Group S_3 cordial sum divisor
n
graph.

Proof. Let G = C_n^2 where n is even and $V(G) = \{u_i : 0 \leq i \leq n-1\}$ and $E(G) = \{u_iu_{i+1} : 0 \leq i \leq n-1\} \cup \{u_iu_{i+2} : 0 \leq i \leq n-2\}$Then G is of order n and size 2n. Define h: $V(G) \rightarrow S_3$ as follows

$$h(v\mathrm{i}) = \begin{cases} a \;\; if \; i \equiv 1 \; (mod \; 6) \; and \; 1 \; \leq i \; \leq n \\ d \; if \; i \equiv 2 \; (mod \; 6) \; and \; 1 \; \leq i \; \leq n \\ c \; if \; i \equiv 3 \; (mod \; 6) \; and \; 1 \; \leq i \; \leq n \end{cases}$$

$$h(v\mathrm{i}) = \begin{cases} e \;\; if \; i \equiv 4 \; (mod \; 6) \; and \; 1 \; \leq i \; \leq n \\ b \; if \; i \equiv 5 \; (mod \; 6) \; and \; 1 \; \leq i \; \leq n \\ f \; if \; i \equiv 0 \; (mod \; 6) \; and \; 1 \; \leq i \; \leq n \end{cases}$$

From **TABLE** 4, we can easily verify that $|v_h(\mathbf{i}) - v_h(\mathbf{j})| \leq 1$ for every $\mathbf{i}, \mathbf{j} \in S_3$ and $|e_h(1) - e_h(0)| \leq 1$ Therefore, C_n^2,is even admits a Group S_3 cordial sum divisor labeling. Hence C_n^2 is even ^{n}is a group S_3 cordial sum divisor graph.

Theorem 2.5. A graph obtained by attaching the central vertex $K_{1,2}$ at each pen-dant vertex of a comb $P_n \odot K_1$ admits Group S_3 cordial sum divisor graph.

Proof. Let G be a graph obtained by attaching the central vertex of $K_{1,2}$ at eachof a Comb.
Let $V(G) = \{u_i, v_i, x_i, y_i : 1 \leq i \leq n\}$ and $E(G) = \{u_iu_{i+1} : 1 \leq i \leq n-1\} \cup \{u_iv_i, v_ix_i, v_iy_i : 1 \leq i \leq n\}$
be the vertex set and edge set of G respectively.Then G is of order 4n and size 4n − 1.
Define h : $V(G) \rightarrow S_3$ as follows

TabLE 5

Nature of n	$v_h(a)$	$v_h(b)$	$v_h(c)$	$v_h(d)$	$v_h(e)$	$v_h(f)$	$e_h(0)$	$e_h(1)$
$3k+1(k \geq 1)$	$2k$	$2k+1$	$2k$	$2k+1$	$2k+1$	$2k+1$	$6k+1$	$6k+2$

$3k+2(k \geq 0)$	$2k+1$	$2k+1$	$2k+2$	$2k+1$	$2k+1$	$2k+2$	$6k+3$	$6k+4$
$3k(k \geq 1)$	$2k$	$2k$	$2k$	$2k$	$2k$	$2k$	$6k-1$	$6k$

$$h(u\mathrm{i}) = \begin{cases} e \ \ if \ i \equiv 1 \ (mod \ 3) \ and \ 1 \ \leq i \ \leq n \\ a \ if \ i \equiv 2 \ (mod \ 3) \ and \ 1 \ \leq i \ \leq n \\ b \ if \ i \equiv 0 \ (mod \ 3) \ and \ 1 \ \leq i \ \leq n \end{cases}$$

$$h(v\mathrm{i}) = \begin{cases} f \ \ if \ i \equiv 1 \ (mod \ 3) \ and \ 1 \ \leq i \ \leq n \\ c \ if \ i \equiv 2 \ (mod \ 3) \ and \ 1 \ \leq i \ \leq n \\ d \ if \ i \equiv 0 \ (mod \ 3) \ and \ 1 \ \leq i \ \leq n \end{cases}$$

$$h(x\mathrm{i}) = \begin{cases} d \ \ if \ i \equiv 1 \ (mod \ 3) \ and \ 1 \ \leq i \ \leq n \\ c \ if \ i \equiv 2 \ (mod \ 3) \ and \ 1 \ \leq i \ \leq n \\ e \ if \ i \equiv 0 \ (mod \ 3) \ and \ 1 \ \leq i \ \leq n \end{cases}$$

$$h(y\mathrm{i}) = \begin{cases} b \ \ if \ i \equiv 1 \ (mod \ 3) \ and \ 1 \ \leq i \ \leq n \\ f \ if \ i \equiv 2 \ (mod \ 3) \ and \ 1 \ \leq i \ \leq n \\ a \ if \ i \equiv 0 \ (mod \ 3) \ and \ 1 \ \leq i \ \leq n \end{cases}$$

From TABLE 5,we can easily verify that $|v_h(\mathbf{i}) - v_h(\mathbf{j})| \leq 1$ for every $\mathbf{i}, \mathbf{j} \in S_3$ and $|e_h(1)-e_h(0)| \leq 1$ Therefore, G admits a Group S_3 cordial sum divisor labeling. Hence G is a group S_3 cordial sum divisor graph.

Theorem 2.6. $S(k_{1,n})$ for all $n \geq 1$, is a Group S_3 cordial sum divisor graph.

Proof. Let $G = S(k_{1,n})$.Let $V(G) = \{u, v_i, u_i : 1 \leq i \leq n\}$ and $E(G) = \{uv_i, v_iu_i : 1 \leq i \leq n\}$ Here G is of order $2n + 1$ and size $2n$. Define $h : V(G) \rightarrow S_3$ as follows
$h(u) = a$

$$h(v\mathrm{i}) = \begin{cases} b \ \ if \ i \equiv 1 \ (mod \ 3) \ and \ 1 \ \leq i \ \leq n \\ c \ if \ i \equiv 2 \ (mod \ 3) \ and \ 1 \ \leq i \ \leq n \\ a \ if \ i \equiv 0 \ (mod \ 3) \ and \ 1 \ \leq i \ \leq n \end{cases}$$

$$h(u\mathrm{i}) = \begin{cases} d \ \ if \ i \equiv 1 \ (mod \ 3) \ and \ 1 \ \leq i \ \leq n \\ e \ if \ i \equiv 2 \ (mod \ 3) \ and \ 1 \ \leq i \ \leq n \\ f \ if \ i \equiv 0 \ (mod \ 3) \ and \ 1 \ \leq i \ \leq n \end{cases}$$

TabLE 6

Nature of n	$v_h(a)$	$v_h(b)$	$v_h(c)$	$v_h(d)$	$v_h(e)$	$v_h(f)$	$e_h(0)$	$e_h(1)$
$6k+1(k \geq 0)$	$2k+1$	$2k+1$	$2k$	$2k+1$	$2k$	$2k$	$6k+1$	$6k+1$
$6k+2(k \geq 0)$	$2k+1$	$2k+1$	$2k+1$	$2k+1$	$2k+1$	$2k$	$6k+2$	$6k+2$
$6k+3(k \geq 0)$	$2k+2$	$2k+1$	$2k+1$	$2k+1$	$2k+1$	$2k+1$	$6k+3$	$6k+3$

$6k+4 (k \geq 0)$	$2k+2$	$2k+2$	$2k+1$	$2k+2$	$2k+1$	$2k+1$	$6k+4$	$6k+4$
$6k+5 (k \geq 0)$	$2k+2$	$2k+2$	$2k+2$	$2k+2$	$2k+2$	$2k+1$	$6k+5$	$6k+5$
$6k (k \geq 1)$	$2k+1$	$2k$	$2k$	$2k$	$2k$	$2k$	$6k$	$6k$

From **TABLE** 6,we can easily verify that $|v_h(\mathbf{i}) - v_h(\mathbf{j})| \leq 1$ for every $\mathbf{i}, \mathbf{j} \in S_3$ and $|e_h(1) - e_h(0)| \leq 1$ Therefore, $S(k_{1,n})$ for all $n \geq 1$ admits a Group S_3 cordial sum divisor labeling. Hence $S(k_{1,n})$ for all $n \geq 1$ is a group S_3 cordial sum divisor graph.

Theorem 2.7. The graph $H_n \odot K_1$ is a group S_3 cordial sum divisor graph.

Proof. Let be a $H_n \odot K_1$ graph with the vertex set $V(H_n) = \{u_i, v_i : 1 \leq i \leq n\}$ and Let $u_1, u_2, u_3, \ldots, u_n$ and $v_1, v_2, v_3, \ldots, v_n$ be the pendant vertices connected to $v_1, v_2, \ldots, v_n$ respectively in G. Then $|V(G)| = 4n$ and $|\mathbf{E}(G)| = 4n - 1$.

Hence the vertices $u_{(n+1)}$ and $v_{(n+1)}$ are connected by an edge, if n is odd or the vertices u_n and $v_{(n)+1}$ by an edge, if n is even. Define $h: V(G) \rightarrow S_3$ as follows

$$h(u\mathrm{i}) = \begin{cases} c \ \ if \ i \equiv 1 \ (mod\ 3) \ and \ 1 \ \leq i \ \leq n \\ a \ if \ i \equiv 2 \ (mod\ 3) \ and \ 1 \ \leq i \ \leq n \\ b \ if \ i \equiv 0 \ (mod\ 3) \ and \ 1 \ \leq i \ \leq n \end{cases}$$

$$h(u'\mathrm{i}) = \begin{cases} f \ \ if \ i \equiv 1 \ (mod\ 3) \ and \ 1 \ \leq i \ \leq n \\ d \ if \ i \equiv 2 \ (mod\ 3) \ and \ 1 \ \leq i \ \leq n \\ e \ if \ i \equiv 0 \ (mod\ 3) \ and \ 1 \ \leq i \ \leq n \end{cases}$$

$$h(v\mathrm{i}) = \begin{cases} a \ \ if \ i \equiv 1 \ (mod\ 3) \ and \ 1 \ \leq i \ \leq n \\ b \ if \ i \equiv 2 \ (mod\ 3) \ and \ 1 \ \leq i \ \leq n \\ c \ if \ i \equiv 0 \ (mod\ 3) \ and \ 1 \ \leq i \ \leq n \end{cases}$$

$$h(v'\mathrm{i}) = \begin{cases} d \ \ if \ i \equiv 1 \ (mod\ 3) \ and \ 1 \ \leq i \ \leq n \\ e \ if \ i \equiv 2 \ (mod\ 3) \ and \ 1 \ \leq i \ \leq n \\ f \ if \ i \equiv 0 \ (mod\ 3) \ and \ 1 \ \leq i \ \leq n \end{cases}$$

It is easy to verify that $|v_h(\mathbf{i}) - v_h(\mathbf{j})| \leq 1$ for every $\mathbf{i}, \mathbf{j} \in S_3$ and $|e_h(1) - e_h(0)| \leq 1$ Therefore, g is a Group S_3 cordial sum divisor labeling.

Theorem 2.8. Globe $Gl(n), n \geq 3$ is a Group S_3 cordial sum divisor graph.

Proof. Let $G = Gl(n)$.Let $V(G) = \{u, v, w_i : 1 \leq i \leq n\}$ and $E(G) = \{[uw_i] \cup [vw_i] : 1 \leq i \leq n\}$.Here,$|V(G)| = n$ and $|\mathbf{E}(G)| = 2n$. Define $h : V(G) \rightarrow S_3$ as follows

$h(u) = a$

$h(v) = f$

$$h(w\mathrm{i}) = \begin{cases} b \ \ if \ i \equiv 1 \ (mod\ 6) \ and \ 1 \ \leq i \ \leq n \\ c \ if \ i \equiv 2 \ (mod\ 6) \ and \ 1 \ \leq i \ \leq n \\ d \ if \ i \equiv 3 \ (mod\ 6) \ and \ 1 \ \leq i \ \leq n \end{cases}$$

$$h(wi) = \begin{cases} e \;\; if \; i \equiv 4 \; (mod\; 6) \; and \; 1 \; \le i \; \le n \\ a \; if \; i \equiv 5 \; (mod\; 6) \; and \; 1 \; \le i \; \le n \\ f \; if \; i \equiv 0 \; (mod\; 6) \; and \; 1 \; \le i \; \le n \end{cases}$$

TABLE 7

Nature of n	$v_h(b)$	$v_h(c)$	$v_h(d)$	$v_h(e)$	$v_h(a)$	$v_h(f)$	$e_h(0)$	$e_h(1)$
$6k+1 (k \ge 1)$	$k+1$	k	k	k	$k+1$	$k+1$	$6k+1$	$6k+1$
$6k+2 (k \ge 1)$	$k+1$	$k+1$	k	$k+1$	$k+1$	$k+1$	$6k+2$	$6k+2$
$6k+3 (k \ge 0)$	$k+1$	$k+1$	$k+1$	k	$k+1$	$k+1$	$6k+3$	$6k+3$
$6k+4 (k \ge 0)$	$k+1$	$k+1$	$k+1$	$k+1$	$k+1$	$k+1$	$6k+4$	$6k+4$
$6k+5 (k \ge 0)$	$k+1$	$k+1$	$k+1$	$k+1$	$k+2$	$k+1$	$6k+5$	$6k+5$
$6k (k \ge 1)$	k	k	k	k	$k+1$	$k+1$	$6k+6$	$6k+6$

From TABLE 7,we can easy verify that $|v_h(\mathbf{i}) - v_h(\mathbf{j})| \le 1$ for every $\mathbf{i}, \mathbf{j} \in S_3$ and $|e_h(1) - e_h(0)| \le 1$ Therefore, g is a Group S_3 cordial sum divisor labeling.

Theorem 2.9. The graph obtained by path union of two copies of Globe graph isa Group S_3 cordial sum divisor graph.

Proof. Consider the two copies of Globe graph $Gl^1(n)$ and $Gl^2(n)$ respectively. Let $G_1 = Gl^1(n)$ and $G_2 = Gl^2(n)$.Let $V(G_1) = \{u, v, w_i : 1 \le i \le n\}$ and $E(G_1) = \{[uw_i] \cup [vw_i] : 1 \le i \le n\}$. Let $V(G_2) = \{u_1, v_1, x_i : 1 \le i \le n\}$ and $E(G_2) = \{[u_1x_i] \cup [v_1x_i] : 1 \le i \le n\}$. Now , Let G be the graph obtained by the path union of copies of Global graphs G_1 and G_2. Here, We add a path of length one from G_1 from G_2. Then $V(G) = V(G_1) \cup V(G_2)$ and $E(G) = E(G_1) \cup E(G_2) \cup E(uu_1)$. Here,$|V(G)| = 2n$ and $|\mathbf{E}(G)| = 4n + 1$. Define $h : V(G) \to S_3$ as follows
$h(u) = g(u_1) = a$
$h(v) = g(v_1) = f$

$$h(wi) = g(xi) = \begin{cases} b \;\; if \; i \equiv 1 \; (mod\; 6) \; and \; 1 \; \le i \; \le n \\ c \; if \; i \equiv 2 \; (mod\; 6) \; and \; 1 \; \le i \; \le n \\ d \; if \; i \equiv 3 \; (mod\; 6) \; and \; 1 \; \le i \; \le n \end{cases}$$

$$h(wi) = g(xi) = \begin{cases} e \;\; if \; i \equiv 4 \; (mod\; 6) \; and \; 1 \; \le i \; \le n \\ a \; if \; i \equiv 5 \; (mod\; 6) \; and \; 1 \; \le i \; \le n \\ f \; if \; i \equiv 0 \; (mod\; 6) \; and \; 1 \; \le i \; \le n \end{cases}$$

TabLE 8

Nature of n	$v_h(b)$	$v_h(c)$	$v_h(d)$	$v_h(e)$	$v_h(a)$	$v_h(f)$	$e_h(0)$	$e_h(1)$
$6k+1(k \geq 1)$	$2k+2$	$2k$	$2k$	$2k$	$2k+2$	$2k+2$	$12k+2$	$12k+3$
$6k+2(k \geq 1)$	$2k+2$	$2k+2$	$2k$	$2k$	$2k+2$	$2k+2$	$12k+4$	$12k+5$
$6k+3(k \geq 0)$	$2k+2$	$2k+2$	$2k+2$	$2k$	$2k+2$	$2k+2$	$12k+6$	$12k+7$
$6k+4(k \geq 0)$	$2k+2$	$2k+2$	$2k+2$	$2k+2$	$2k+2$	$2k+2$	$12k+8$	$12k+9$
$6k+5(k \geq 0)$	$2k+2$	$2k+2$	$2k+2$	$2k+2$	$2k+4$	$2k+2$	$12k+10$	$12k+11$
$6k(k \geq 1)$	$2k$	$2k$	$2k$	$2k$	$2k+2$	$2k+2$	$12k+12$	$12k+13$

From TABLE 8,we can easily verify that $|v_h(\mathbf{i}) - v_h(\mathbf{j})| \leq 1$ for every $\mathbf{i}, \mathbf{j} \in S_3$ and $|e_h(1) - e_h(0)| \leq 1$. Therefore, h is a Group S_3 cordial sum divisor labeling.

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