Magnetic Field Effect on Oscillatory Couette Flow Regime

Pradeep Kr Singh, Rajan Kr Sharma, Anil Kr Trivedi

Department of Mathematics, Pranveer Singh Institute of Technology, Kanpur, Uttar Pradesh, India.

Department of Mathematics, Firoz Gandhi Institute of Engineering and Technology,

Raibarelli, Uttar Pradesh, India.

Abstract:

It has been observed that there is a lot of natural and industrial flows which are dependent with different boundaries condition with time. In this presented paper, of a quantitative analysis of oscillatory MHD Couette flow has been performed by using perturbation technique. In ideal flow, the magnetic field is fixed function of radius with two parameters only: a ratio of inner to outer cylinder radii and a ratio of the magnetic field values on outer and inner cylinders in an incompressible fluid. The effect of magnetic field on the flow of an electrically conducting viscous fluid has received considerable attention due to its wide range of engineering, geophysical, astrophysical applications and in oscillatory free convective flow problem in the presence of a magnetic field through a porous medium have attracted the attention of a number of researchers. The unsteady laminar boundary layer flow of an electrically conducting viscous fluid near an impulsively started flat plate of infinite extent is considered, with a view to examine the influence of transverse magnetic field fixed to the fluid. It has been found that the amplitude of the unsteady velocity gradient is smaller at upper plate in comparison to lower plate for small harmonic oscillations. Variation of the amplitude and phase lag of the shear stress at lower wall with respect to magnetic parameter for fixed value of frequency parameter. The unsteady velocity gradient increases with respect to applied magnetic field.

Key Words: MHD, Harmonic Oscillation, Rate of heat transfer, Magnetic field, Oscillatory flow.

1. Introduction

The problem of time-dependent oscillatory flow in which fluctuating part of the motion is superimposed on mean steady parts was studied by various research group Lighthill, Stuart and Ishigaki, Soundalgekar, Acharya et al. In fluid dynamics, Couette flow (1) is the flow of a viscous fluid in the space between two surfaces, one of which is moving tangentially relative to the other. The relative motion of the surfaces imposes a shear stress on the viscus fluid.

The magnetic analogue of Lighthill's analysis was done by Ahmed and Soundalgekar et al. Their investigation led to an increase of the oscillatory flow-based research on small harmonic oscillation in a channel of gap 'h'. The analysis based on Ishigaki work in which he assumed unsteady flow of the stream with its conjugate complex part. The following observation was obtained such as, the amplitude of the fluctuating velocity gradient varies with respect to transverse magnetic field (Light hill and M. J 1954, Soundalgekar et al 2001, T. J Stuart & Ishigaki hahzed 1955, M Acharya et al. 2008, S. Ahmed & N. Ahmed 2004, B V Soundalgekar 1973).

In recent years, oscillatory free convective flow and heat transfer problem in the presence of a magnetic field through a porous medium (4) have attracted the attention of many researchers, due to their possible application in many branches of science and technology. In practice, cooling of porous structure is achieved by forcing the liquid or gas through capillaries of solid. Actually, they were used insulate a heated body to maintain its temperature. Study of origin of flow through a porous medium is heavily based on Darcy's experimental law. In view of Darcy's law, Yamamoto K and Yoshida, Hooper et al, considered suction and injection flow with convective acceleration through a plane porous wall especially for the flow outside a vertex layer. Yamamoto K and Iwamura have presented the generalization of the above study. The oscillatory flow past a porous bed studied by Chawla S and Singh S, Raptis et al, have presented the study of two-dimensional flow of viscous fluid through a porous medium bounded by a porous surface subject to a constant suction velocity by taking account of free convection currents (W. B Hooper et al. 1994, Yamamoto K and Yashida Z 1974, Yamamoto K and Iwamura N 1976, A. Rapits et al. 1981, Chawla S et al 1979).

2. Mathematical Formulation

We consider the two – dimensional flow of an incompressible fluid with constant properties and magnetic field between two parallel flat walls, one which is at rest and other moving its own plane with an unsteady velocity as shown in Figure 1. We restricted our consideration to the case in which the flow is independent of the distance along the wall and the velocity component normal wall is zero.

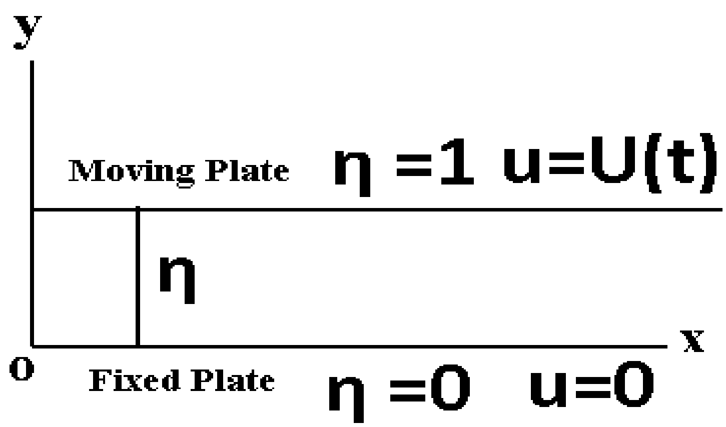

Figure 1. Systematic of two-dimensional flow of an incompressible fluid.

We have neglected inertial force in comparison to viscous force in Navier-Stokes equation of motion. In Couette flow motion is induced due to the movement of the upper plate.

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \vartheta\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u \quad (1)$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} \quad (2)$$

The fixed plate is along x-axis and y-axis is normal to it. Plates are at distance 'h' apart. The expression of unsteady velocity between the plates is given by,

$$\boldsymbol{U(t) = U_h\left[1 + \frac{t}{2}(e^{i\omega t} + e^{-i\omega t})\right]} \quad (3)$$

Where, ω is the frequency parameter and $U_h$and t are constants.We considered the solution of equation (1) as

$$u = U_h\left[u_0(\eta) + \frac{t}{2}\{u_1(\eta)e^{i\omega t} + \overline{u_1}(\eta)e^{-i\omega t}\}\right] \quad (4)$$

Where, $\eta = \frac{y}{h}$ On substituting equation (3) and (4) in equation (1) and equating steady and unsteady parts, we have,

$$u_0'' = 0 \qquad (5)$$

$$u_1'' - (m + ik)u_1 = -(m + ik) \qquad (6)$$

The boundary conditions are,

$$\eta = 0,\ u_0 = u_1 = 0 \text{ and } \eta = 1, u_0 = u_1 = 1 \qquad (7)$$

Where, $m = \frac{\sigma B_0^2}{\rho}\frac{h^2}{\nu}$ and $k = \frac{\omega h^2}{\nu}$ are magnetic and frequency parameters respectively.

Equations (5) and (6) are simple linear differential equations, whose complementary and particular solution constituted as :

The solution of equation (5) is

$$u_0 = \eta \qquad (8)$$

The solution of equation (6) is,

$$u_1 = 1 - cosh(m + ik)^{\frac{1}{2}}\eta + coth(m + ik)^{\frac{1}{2}} sinh(m + ik)^{\frac{1}{2}}\eta \qquad (9)$$

The magnitude of the fluctuating velocity gradient at lower wall is,

$$|\ u_1'\ | = \frac{2}{(cosh2\theta_1 - cos2\theta_2)}(\theta_1^2 + \theta_2^2)^{\frac{1}{2}}(sinh^2 2\theta_1 + sin^2 2\theta_2)^{\frac{1}{2}} \qquad (10)$$

The magnitude of the fluctuating velocity gradient at mid plane is,

$$|\ u_1'\ | = \frac{2}{(cosh2\theta_1 - cos2\theta_2)}(\theta_1^2 + \theta_2^2)^{\frac{1}{2}}(sinh^2\theta_1 cos^2\theta_2 + cosh^2\theta_1 sin^2\theta_2)^{\frac{1}{2}} \times$$

$$(cosh^2 0.5\theta_1 cos^2 0.5\theta_2 + sinh^2 0.5\theta_1 sin^2 0.5\theta_2)^{\frac{1}{2}} \quad (11)$$

The magnitude of the fluctuating velocity gradient at upper wall is,

$$| u_1' | = \frac{2}{(cosh2\theta_1 - cos2\theta_2)} (\theta_1^2 + \theta_2^2)^{\frac{1}{2}} (sinh^2\theta_1 cos^2\theta_2 + cosh^2\theta_1 sin^2\theta_2)^{\frac{1}{2}} \quad (12)$$

Where $\theta_1 = (m^2 + k^2)^{\frac{1}{4}} cos\alpha$, $\theta_2 = (m^2 + k^2)^{\frac{1}{4}} sin\alpha$ and $\alpha = \frac{1}{2} tan^{-1} \frac{k}{m}$

The shear stress at $y = 0$ is given by,

$$T_\omega = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$T_\omega = \frac{\mu U_h}{h} \left\{1 + \frac{t}{2}\left(Ae^{i\omega t} + \overline{A}e^{-i\omega t}\right)\right\} \quad (13)$$

Where, $A = (m + ik)^{\frac{1}{2}} \coth(m + ik)^{\frac{1}{2}}$ (14)

An expression of the amplitude of shear stress of lower wall is given by,

$$| A | = \frac{1}{(cosh2\theta_1 - cos2\theta_2)^{\frac{1}{2}}} (\theta_1^2 + \theta_2^2)^{\frac{1}{2}} (sin^2 2\theta_1 + sinh^2 2\theta_2)^{\frac{1}{2}} \quad (15)$$

An expression of the phase angle of shear stress of lower wall is given by,

$$\psi = tan^{-1} \left\{\frac{\theta_2 sin2\theta_1 - \theta_1 sinh2\theta_2}{\theta_1 sin2\theta_1 + \theta_2 sinh2\theta_2}\right\} \quad (16)$$

2.1 Temperature Field

Under the flow conditions, the equations for the temperature distribution with negligible heat generation due to Joule's effect is,

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = K\left\{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right\} + \frac{\nu}{c}\left(\frac{\partial u}{\partial y}\right)^2 \quad (17)$$

In this case, x-denotes the distance along the wall, T is the temperature, K is the thermal diffusivity and C is the specific heat.

The following boundary condition were used to solve Equation (17),

$$T = T_W \quad \text{or} \quad \frac{\partial T}{\partial y} = 0 \quad \text{at} \quad y = 0$$

$$\text{At} \quad y = h, \; T = T_h \quad (18)$$

With these boundary conditions the solution obtained is independent of x. In the present case heat transferred due to convection is negligible. Temperature distribution is caused only due to conduction and viscous dissipation.

Thus, we have,

$$\frac{\partial T}{\partial t} = K\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{C}\left(\frac{\partial u}{\partial y}\right)^2 \quad (19)$$

Looking for the solution of equation (19) in the form as,

$$T(\eta) = T_0(\eta) + \frac{1}{2}\epsilon\left\{T_1(\eta)e^{i\omega t} + \overline{T_1}(\eta)e^{-i\omega t}\right\} + \frac{1}{2}\epsilon^2\left[T_2(\eta)e^{2iwt} + \overline{T_2}(\eta)e^{-2i\omega t}\right]$$

$$(20)$$

Equating the harmonic coefficients to zero, we have

$$T_0'' = -\frac{P_r U_h^2}{C} \quad (21)$$

$$T_1'' - iP_r k T_1 = -\left(\frac{P_r\, U_h^2}{C}\right) u_0' u_1' \quad (22)$$

Where $P_r = \frac{\mu C}{k}$ is the Prandtle number, $k = \frac{\omega h^2}{\nu}$ is frequency parameter.

The boundary conditions reduce to as,

$$T_0 = T_\omega, \quad T_1 = T_2 = 0 \quad \text{at} \quad \eta = 0$$

$$T_0 = T_h, \quad T_1 = T_2 = 0 \quad \text{at} \quad \eta = 1 \tag{23}$$

The solution of equation (22) and (23) are

$$\frac{T_0 - T_h}{T_\omega - T_h} = -P_r E \frac{\eta}{2}(\eta - 1) + \eta + 1 \tag{24}$$

$$\frac{T_1}{T_\omega - T_h} = P_r E \frac{l}{(l^2 - I^2)}[sinhl\eta(cosechI\ coshl + 1)$$

$$+ cothl\{cosechIsinhI(1-\eta) - coshl\eta\}] \tag{25}$$

Where, $l^2 = m + ik$, $I^2 = iP_r k$ and $E = \frac{U_h^2}{C(T_w - T_h)}$, Eckert number

The rate of heat transfer

$$\frac{T_1'(\eta)}{T_\omega - T_h} = P_r E \frac{l}{(l^2 - I^2)}\left[\frac{coshl\eta}{l}(cosechIcoshl + 1)\right.$$

$$\left. - cothl\left\{cosechIcoshI\frac{(1-\eta)}{I} + \frac{sinhl\eta}{l}\right\}\right] \tag{26}$$

The rate of heat transfer at lower plate is given by,

$$\frac{T_1'(0)}{T_\omega - T_h} = \frac{P_r E}{I(l^2 - I^2)}[I + cosechIcoshl(I - lcosechlcoshI)] \tag{27}$$

The rate of heat transfer at upper plate is given by,

$$\frac{T_1{}'(1)}{T_\omega - T_h} = \frac{P_r E}{I(l^2 - I^2)} \frac{cothl}{2sinhI} [Isinh2l - 2l] \tag{28}$$

3. Result and Discussion

The distribution of the magnitude of fluctuating velocity gradient $|u_1{}'|$ are shown in table 1 for several values of m and at fixed value of frequency parameter. In figure 1, at ɳ=0 the fluctuating velocity gradient increases and then decreases and suddenly increases on increasing m and again constant and again increases. But at ɳ=0.5, the fluctuating velocity gradient increases on increasing m and at ɳ=1, fluctuating velocity gradient slow down and again increases on increasing m. Velocity gradient is smaller at upper plate in comparison to that at lower plate. Variation of the amplitude and phase lag of the shear stress at lower wall with respect to magnetic parameter for fixed value of frequency parameter has been shown in table2 and in figure 2 and 3.The amplitude of shear stress decreases rapidly between m=1 and m=2, then after it becomes constant with magnetic parameter. In table 2 and figure 3 phase lag of the shear stress increases on increasing magnetic field parameter. Several values of frequency parameter for lower wall with fixed value of magnetic field parameter has been shown in table3 and figure 4 and 5. We observe that amplitude of the shear stress at lower plate increases with increasing frequency parameter for fixed magnetic field parameter are shown in figure 4. Moreover, we observe that the phase lag decreases with increasing frequency parameter as shown in Figure 5.

The amplitude and phase lag of rate of heat transfer at lower plate increases with increasing magnetic field parameter. However, the phase lag of rate of heat transfer at upper plate is same at lower plate. These variations are shown in table 4 and 5and also shown in figure 6, 7, 8 and 9.

Table 1: Variations of fluctuating velocity gradient with magnetic field. For fixed value K=1

η \ m	$|u_{1'}|$		
	0.0	**0.5**	**1.0**
0	0.09	0.25	0.20
1	2.70	0.61	0.52
2	3.33	0.94	0.73
3	3.79	1.15	0.83
4	4.20	1.98	0.95
5	5.01	2.32	1.02
6	5.98	3.01	2.31
7	6.02	3.99	3.12
8	6.99	4.20	4.05
9	7.73	5.39	5.13
10	8.08	6.17	6.09

Table 2: Variations of amplitude and phase lag of the shear stress at lower wall. For fixed value K=2

m	$|A|$	**ψ**
1	43.2995	31.9641
2	4.8000	49.2559
3	1.8000	53.9830
4	1.6000	61.3257

5	1.6000	74.5323

Table 3: Variations of amplitude and phase lag of the shear stress at lower wall. For fixed value m =2

K	$\|A\|$	**ψ**
1	0.3880	72.8051
2	1.5694	60.6989
3	2.3435	52.8321
4	4.3413	49.3235
5	5.8898	40.3135

Table 4: Heat transfer variation at lower wall with respect to magnetic field parameter m.

P_r=2,E=10,K=5

m	$\frac{T'_1(0)}{T_w - T_h}$	$\|\beta_1\|$	$\emptyset_1$
1	9.2631- 1.5032i	9.3842	10.2416
5	5.2140+ 10.5660i	11.7824	70.8167
10	1.2315+ 15.3216i	15.3710	94.8940

Table 5: Heat transfer variation at upper wall with respect to magnetic field parameter m.

P_r=2, E=10, K=5

m	$\frac{T'_1(1)}{T_w - T_h}$	$\|\beta_2\|$	$\emptyset_2$
1	7.6567- 0.3548i	7.6649	2.9478
5	10.5204+ 8.6241i	13.6034	43.7146
10	14.5109+ 13.0990i	19.5486	46.7473

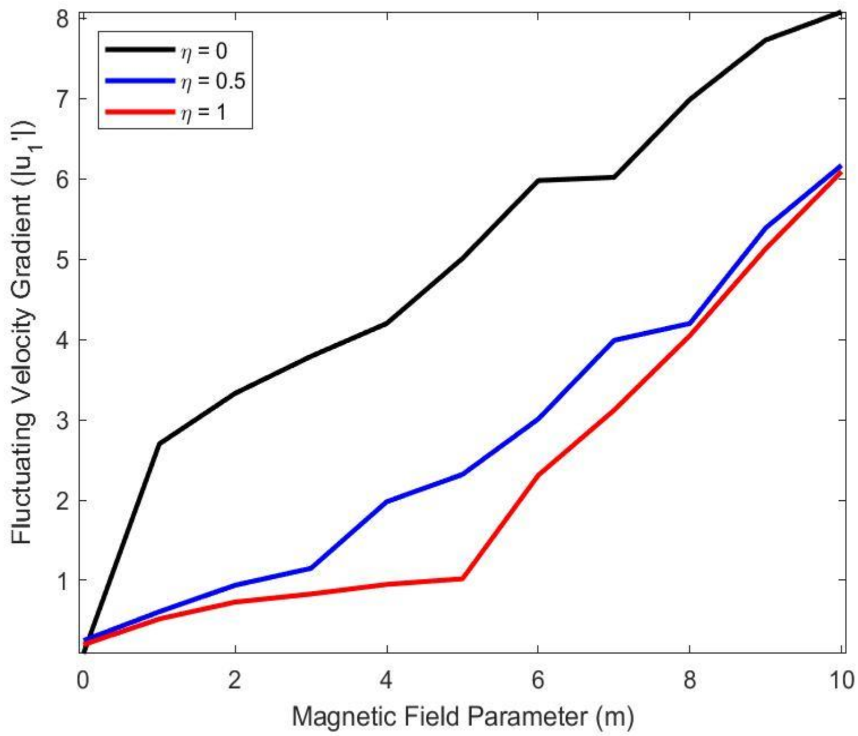

Fig1: Variations of Fluctuating Velocity Gradient with Magnetic Field Parameter

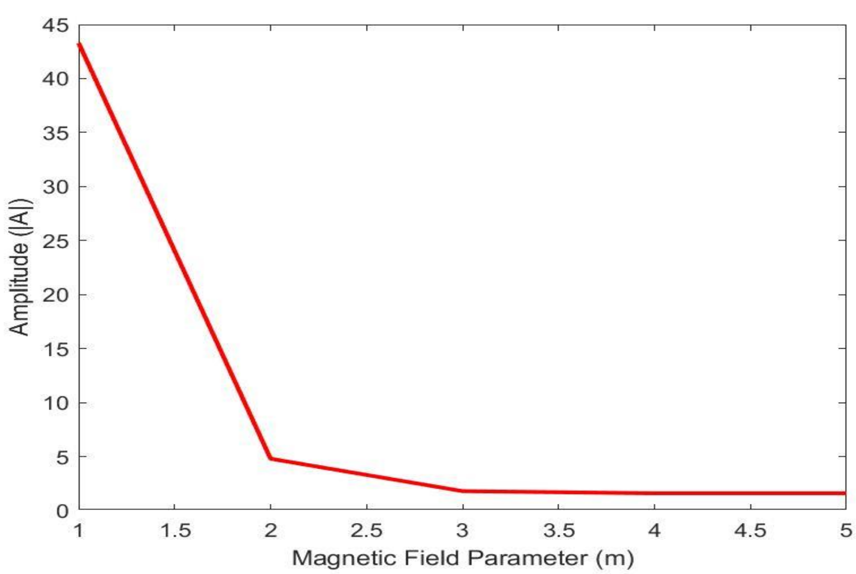

Fig2: Variation of Amplitude with Magnetic Field Parameter

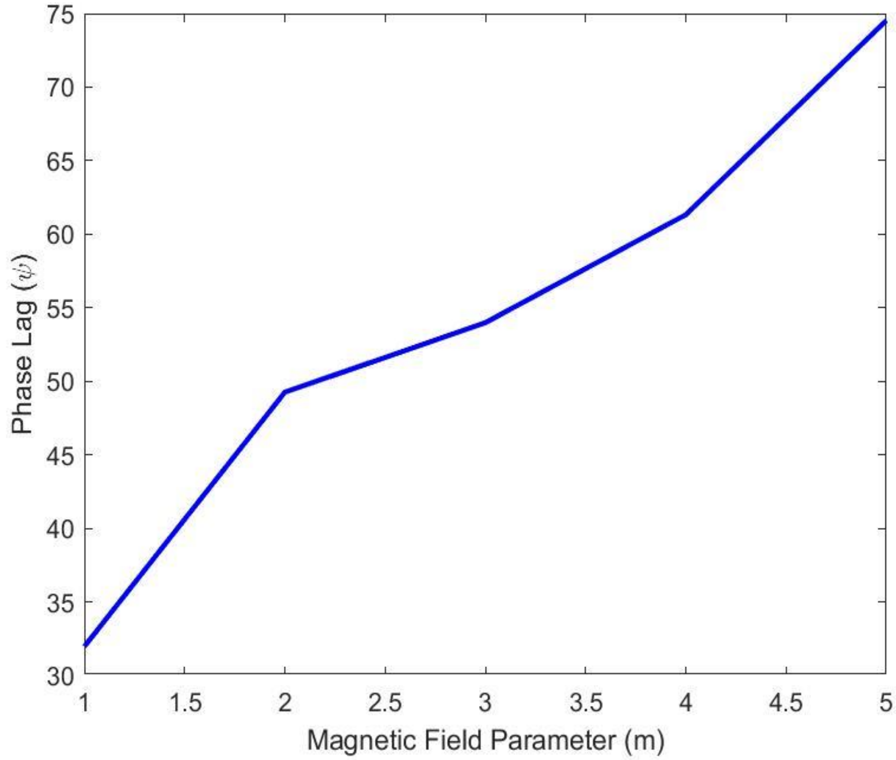

Fig3: Variation of Phase Lag with Magnetic Field Parameter

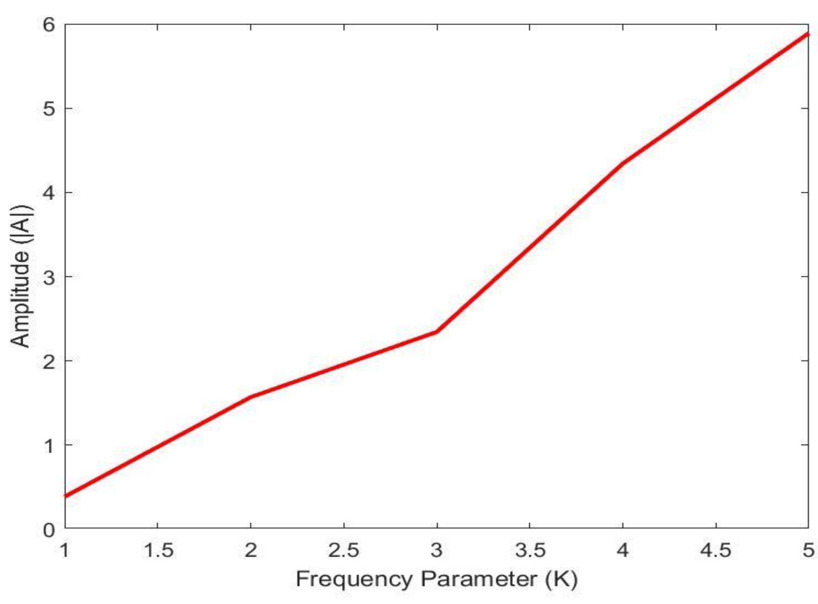

Fig4: Variation of Amplitude with Frequency Parameter

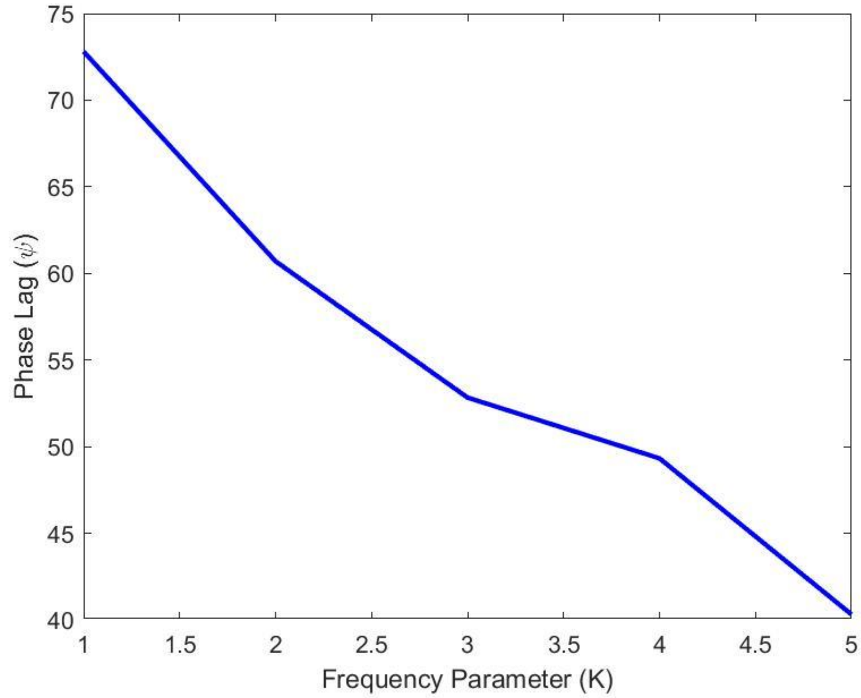

Fig 5: Variation of Phase Lag with Frequency Parameter

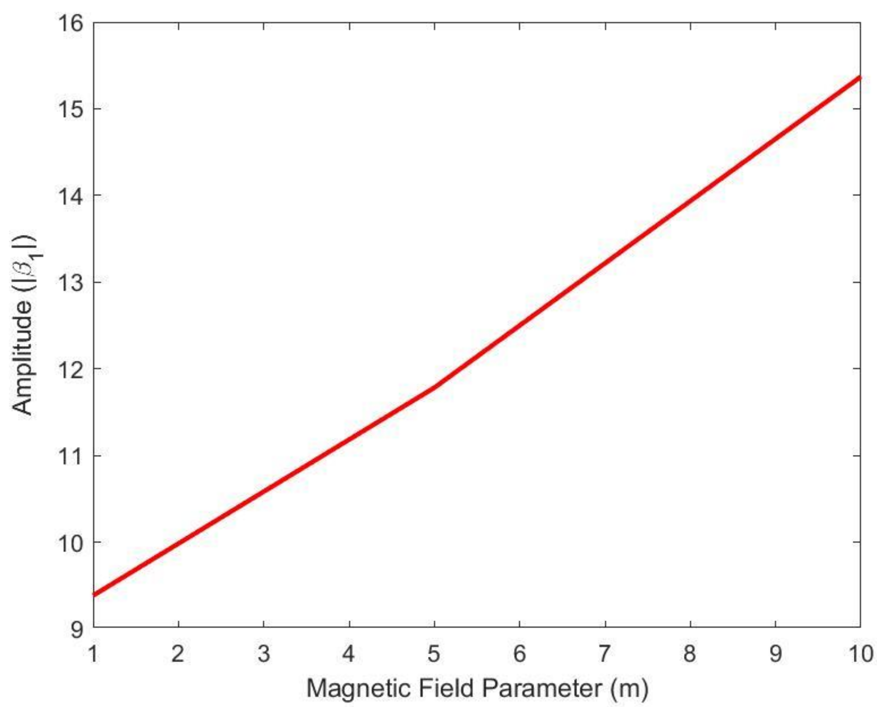

Fig 6: Variation of Amplitude with Magnetic Field Parameter

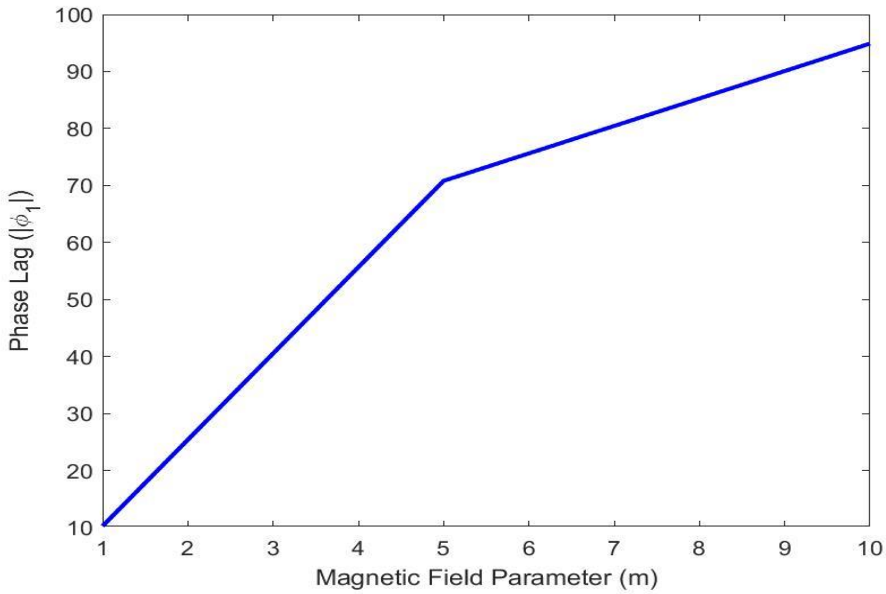

Fig 7: Variation of Phase Lag with Magnetic Field Parameter

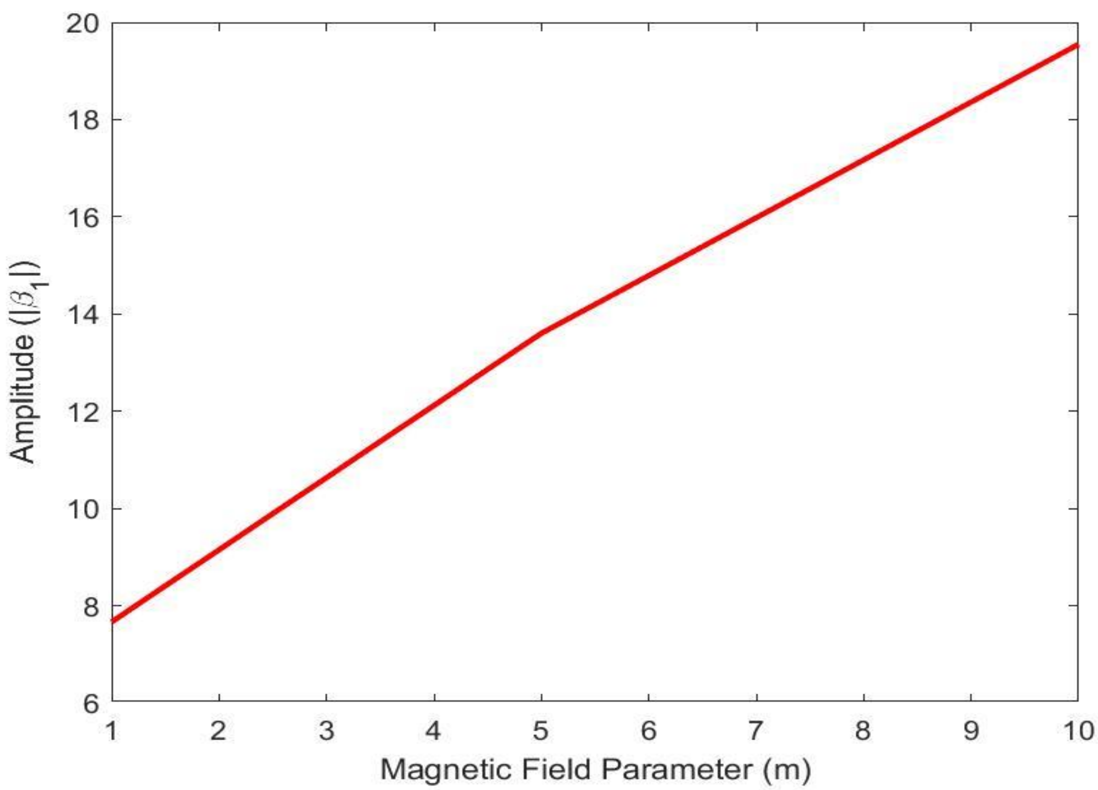

Fig 8: Variation of Amplitude with Magnetic Field Parameter

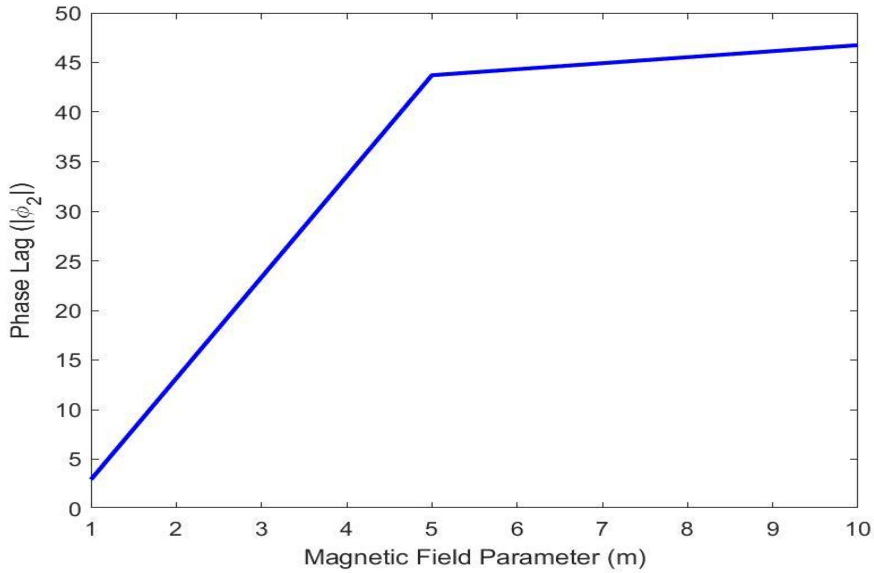

Fig 9: Variation of Phase Lag with Magnetic Field Parameter

References

[1]. Sasebastian Altmeyer: The rigid of instability in ferrofluidic couette flow via alternative magnetic field published. Journal of Scientific report published 25 February 2021,vol11, Article number: 4705

[2]. Reindl, M. and Odenbach, S: Effect of axial and transverse magnetic fields on the flow behavior of ferrofluids featuring different levels of interparticle interaction. *Phys. Fluids* **23**, 093102 (2011).

[3]. Acharya M, Dash C G, Singh P L (2008): "Magnetic field effects on the free convection mass transfer flow through porous medium with constant suction and constant heat flux". Indian J. Pure Appl. Math, Vol. 32 No. 1, pp 1-18.

[4]. Ingham, D.B. and Pop, I.(2005): Transport Phenomena in Porous Medium

[5]. Ahmed S, Ahmed N (2004): "The two –dimensional MHD oscillatory flow along a uniformly moving infinite vertical plate in a porous medium". Journal of Rajasthan Acad. Phy. Sci., Vol 2 No.2, pp. 75-8.

[6] Chawla S and Singh, S. (1979): "Oscillatory flow a part porous bad".Acta. Mech., Vol. 40, pp 205-203.

[7]. Hooper B W, Chen S T, Armaly FB(1964). "The problem of mixed convection along an isothermal vertical plate on porous medium with injection and suction". Number, heat transfer part A, Vol. 25, pp. 317-329.

[8]. Lighthill J.M. (1934): "The response of laminar skin friction and heat transfer to fluctuation in the stream velocity". Proc. Roy. Soc. Land., A22, pp 1-23.

[9]. Rapits A, PerodikisC, Tzivanidis G(1980),: " The steady two dimensional flow of viscous fluid through a porous medium bounded by a porous surface with constant suction ". J .Phys. Vol.14 pp 99- 102.

[10]. Soundalgekar, V.M,Wavarl P , TakherH.S (2001): "Effect of free convection currents on oscillatory flow of an electrically conducting fluid past stationary moving vertical plate with variable suction and constant heat fluctuation." Jntl. J. App. Mech. and Engg. Vol. 6, pp 639-658.

[11]. Stuart T J, Ishigaki hahzed (1955): "A Solution of the Navier- Stokes and energy equation illustrating the response of skin friction and temperature of infinite plate thermometer to fluctuations in the stream velocity". Proc. Rac.Soc. Lond. 955, A-231, pp. 116-130.

[12]. Soundalgekar B V (1973): "Free convection effects on the oscillatory flow past on infinite vertical porous plate with constant suction". Proc. Roy. Soc. Lond. A 333, pp. 25-26.

[13]. Yamamoto K,Yashida Z (1974): Suction and injection flow with convective acceleration through a plate porous wall specifically". J.Phy. Soc.Japan, Vol. 37, pp 774-779.

[14]. Yamamoto K, Iwamura N(1976): "Flow with convective acceleration through a porous medium". J. Engg. Math, Vol.10.pp 41-54.