Projective change between two special(α, β)-Finslermetrics

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Abstract:

In the present paper, we observe the study of projective change between two Special(α, β)-metrics,$L = C_1 + C_2\beta + \frac{\beta^2}{\alpha}$ is a regular Finsler metric if and only if 1-form β satisfies the condition $\beta_x < 1/2$ for any value of x belongs to M.

Keywords:(α, β)-metric,special(α, β)-metric,regularmetric,1-form,projectivechange.

Introduction:

Finslerspaces are the manifolds whose tangent spaces convey anorm which varies easily withthebasepoint.ThegermsofaFinslerspacemaybeobservedintheepoc-makinglectureofB.RiemanndeliveredatGottingenin1854.He laid down the foundation of amore general geometry which was later on called as Riemannian geometry in his honor. But he was not confident about the geometrical explanations of the results in these spaces. Mathematicians neglected the study of such spaces for more than 60 years. A young German Paul Finsler who was hardly of twenty four years, took up the problem related to the space equipped with the metric function whose study was discouraged by Riemann.

In 1918, he submitted his epoc-making thesis to Gottingen University. Thisthesisdrawntheattentionofmostofthemathematiciansworkingingeometry.In 1951, a young German geometer H. Rund introduced a new concept of parallelism considering Finsler geometry as locally Minkowskian. Later on Mokoto Matsumoto and his pupils devoted to such approach and contributed much to this field. In1970, he also organized a symposium on‘The models of Finsler areas many mathematicians such as S.S.Chern,D.Bao,Z.Shen,R.L.Bryant,D.Burogo and S.Ivanow have been working in this field.

TheFinlsergeometrywhichwasthoughtasoflittleuseinthestudyofPhysicalproblems,suddenlygot its applications in the theory of electron microscope. This was demonstrated by a Polish physicist R. S.Ingarden, then several mathematicians are working on the applications of Finsler geometry.P.L.Antonellihas contributed significantly in Biological Sciences and some Finslerians have been used the theory of Finslerspacesin numereous fields of Physics and Biology such as thermodynamics, optics, ecology, evolution and developmental biology.

In India, many mathematicians also contributed to Finsler geometry significantly. Some of them are R. S.Mishra, R. N. Sen, U. P. Singh, B. B. Sinha, H. D. Pande, R.B. Misra, R. S. Sinha, S. D. Dubey, P. N.Pandey, B. N. Prasad, C. S. Bagewadi, S. K. Narasimhamurthy, S. B. Pandey, A.K. Singh, T.N.Pandey,M.K.Guptaand C.K.Mishra.etc,.

Now, we shall discuss some basic concepts of Finsler geometry, which are necessaryforthediscussionsin theremaining chaptersof thethesis.

1. Finslerspace:

1.1. DifferentialManifoldsandExamples

Ann-dimensional differentiable manifold may beaset Mcollectively with circle of relatives of injective maps

$fi\ : Ui \subset R^n \rightarrow fi(Ui) \subseteq M of open stes Ui in R^n$ into Mspecified

1. $\cup i fi(Ui) = M$,
2. For every pair i,j with $fi(Ui) \cap fj(Uj) = W \neq \phi$, the sets $f_j^{-1}(W) and f_j^{-1}(W) are open sets in R^n$ and $f_i^{-1} \circ f_j, f_j^{-1} \circ f_i$ are differentiable,
3. The circle of relatives (Ui, fi) is maximal relative to atleast one and a couple of

Examples

1. R^n is an n-dimensional differentiable manifold.
2. Let S^n be the quality standard unit sphere in R^{n+1} defined as $S^n = \{\xi = (\xi^i) \in R^{n+1}: |\xi| = \sqrt{(\xi^i)^2 = 1}\}$ is n-dimensional differentiable manifold.

1.2. Finsler space Definitions and Examples:

A formal definition of a Finslerspace is as follows:

Definition 1.2.1: *Let M^n be an n-dimensional smooth manifold and $L(x, y)$ be a fundamental functionwhichsatisfiesthe subsequentconditions.*

i) $L(x, y) > 0, for (x, y) \in D,$

ii) $L(x, \lambda y) = |\lambda| L(x, y), for\ any (x, y) \in D and \lambda \in R such that\ (x, \lambda y) \in D,$

iii) *The d– tensor field* $gij(x, y) = \frac{1}{2} \dot{\partial}_i \dot{\partial} j L^2(x, y) \in D$

then $F^n = (M^n, L)$ *istermedFinslerspaceprepared with a essential characteristic* $L(x, y)$ *on* M^n*,while* $\dot{\partial}_i$ *isnon-degenerateonD.*

Definition1.2.2: *Two FinslermetricsLand$\bar{L}$are projectively associated if and only if their spray coefficients have the relation*

$$G^i = \overline{G}^i + P(y) y^i. \tag{1.1}$$

Definition 1.2.3: *A Finsler metric is projectively associated with any other metric in the event that they have the samegeodesicsaspointsets.InRiemanniangeometry,twoRiemannianmetricsareprojectivelyrelatedif their spray coefficients have the relation*

$$G^i{}_\alpha = G^i{}_\alpha + \lambda_{x^k} y^k y^i. \tag{1.2}$$

Definition1.2.4: *Let* $F^n = (M^n, L)$ *and* $\bar{F}^n = (M^n, \bar{L})$ *betwoFinslerspaces on a common underlying manifold* M^n*.*

The relation among the geodesic coefficients G^i of L and geodesic coefficients G_α^i of is given by way of

$$G^i = G_\alpha^i + \alpha Q s_0^i + \{-2Q\alpha s_0 + r_{00}\}\{\psi b^i + \Theta \alpha^{-1} y^i\} \tag{1.3}$$

where,

$$\Theta = \frac{\phi\phi' - s(\phi\phi'' + \phi'\phi')}{2\phi((\phi - s\phi') + (b^2 - s^2)\phi'')} \quad ;$$

$$Q = \frac{\phi'}{\phi - s\phi'}$$

$$\Psi = \frac{1}{2}\frac{\phi''}{\phi((\phi - s\phi') + (b^2 - s^2)\phi'')}$$

2. Curvature Properties of two special $(\boldsymbol{\alpha}, \boldsymbol{\beta})$-metrics

In this segment, we find the projective relation between unique(α, β)-metrics,$L = C_1 + C_2\beta + \frac{\beta^2}{\alpha}$ The geodesic coefficients aregiven by means of with the metric as follows:

$$\theta = \frac{c_1c_2 - 4c_2s + c_2s^2 - 4s^3}{c_1^2 + 2c_1b^2 + 2c_2b^2s + 2s^2(b^2 - c_1) - 3c_2s^3 - 3s^4} \quad (2.1)$$

$$Q = \frac{c_2 + 2s}{c_1 - s^2} \quad (2.2)$$

$$\Psi = \frac{1}{(c_1 + 2b^2) - 3s^2} \quad (2.3)$$

Substituting(2.1),(2.2)and(2.3)into(1.2),we get

$$G^i = G^i_\alpha + \frac{\alpha^2}{c_1\alpha + 2\beta}s^i_0 + \{-2\alpha^2 s_0 + r_{00}\}\left[\frac{\alpha^2}{(c_1 + 2b^2)\alpha^2 - 2\beta^2}b^2\right] + \left[\frac{(c_1c_2 - 4c_2\alpha^2\beta + c_2\alpha^2\beta^2 - 4\alpha\beta^3)y^i}{(c_1^2 + 2c_1b^2)\alpha^4 + 2c_2b^2\alpha^3\beta + 2(b^2 - c_1)\alpha^2\beta^2 - 3c_2\alpha\beta^3 - 3\beta^4}\right] \quad (2.4)$$

We now deduce the following theorem:

Theorem2.2.1.*TheFinslermetric$L = C_1 + C_2\beta + \frac{\beta^2}{\alpha}$isprojectively associated with to$\bar{L} = \bar{\alpha} + \bar{\beta}$ifand simplest if the subsequent situations are satisfied*

$$G^i_\alpha = G^i_\alpha + \theta y^i - \tau a^2 b^i \quad (2.5)$$

$$b_{i|j} = \tau\left[(-1 + 2b^2)a_{ij} - 3b_ib_j\right], \quad d\bar{\beta} = 0 \quad (2.6)$$

Proof: Let us prove the vital condition. Consistent with lemma2.1.1,wegetboth Land$\bar{L}$areDouglasmetrics. Due to the fact, $\bar{L} = \bar{\alpha} + \bar{\beta}$isaDouglasmetric is most effective if

$$b_{i|j} = \tau\left[(-1 + 2b^2)a_{ij} - 3b_ib_j\right] \quad (2.7)$$

For some scalar function$\tau = \tau(x)$,where$b_{i|j}$denote the coefficients of the covariant derivatives of$\beta = b_I y^I$with respect toα. In this case,$\beta$is closed.

By using (2.6) , we have $r_{00} = r\ [(-1 + 2b^2)\ \alpha^2 - 3\beta^2]$.SubstitutingalltheseinGeodesiccoefficientsequationswe

$$G^i_\alpha\left[\frac{(c_1c_2 - 4c_2\alpha^2\beta + c_2\alpha^2\beta^2 - 4\alpha\beta^3)y^i}{(c_1^2 + 2c_1b^2)\alpha^4 + 2c_2b^2\alpha^3\beta + 2(b^2 - c_1)\alpha^2\beta^2 - 3c_2\alpha\beta^3 - 3\beta^4}\right] \quad (2.8)$$

Since$L$isprojective to$\bar{L}$

$$\left[P+\left(\frac{(c_1c_2-4c_2\alpha^2\beta+c_2\alpha^2\beta^2-4\alpha\beta^3)y^i}{(c_1^2+2c_1b^2)\alpha^4+2c_2b^2\alpha^3\beta+2(b^2-c_1)\alpha^2\beta^2-3c_2\alpha\beta^3-3\beta^4}\right)\right]y^i=G \quad (2.9)$$

where$G=G^i-G^i_\alpha+r+r\alpha^2b^i$

Note that the RHS of the above equation is a quadratic form. Then there have to be a one form $\theta=\theta_iy^i$onM,such that

$$\left[P+\tau\left(\frac{(c_1c_2-4c_2\alpha^2\beta+c_2\alpha^2\beta^2-4\alpha\beta^3)y^i}{(c_1^2+2c_1b^2)\alpha^4+2c_2b^2\alpha^3\beta+2(b^2-c_1)\alpha^2\beta^2-3c_2\alpha\beta^3-3\beta^4}\right)\right]=\theta \quad (2.10)$$

Thus,(2.9)becomes

$$G^i_\alpha=G^i_{\bar\alpha}+\theta y^i-\tau\alpha^2b^i \quad (2.11)$$

Equations (2.6) and (2.7) together with (2.11) hence the proof of the necessity. We know that β is closed, it is sufficient toshow,Lisprojectivelyassociated to $\bar\alpha$

From(2.8)and(2.11),wehave

$$G^i_\alpha=G^i_{\bar\alpha}\left[\tau\left(\frac{(c_1c_2-4c_2\alpha^2\beta+c_2\alpha^2\beta^2-4\alpha\beta^3)y^i}{(c_1^2+2c_1b^2)\alpha^4+2c_2b^2\alpha^3\beta+2(b^2-c_1)\alpha^2\beta^2-3c_2\alpha\beta^3-3\beta^4}\right)\right] \quad (2.12)$$

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That is,Lisprojectively related to$\bar\alpha$

Conclusion:

In this paper, we studied the curvature houses of two(α, β)-metrics and projective change among them.

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